VORTICITY TRANSPORT IN RIVLIN-ERICKSEN FLUID IN THE PRESENCE OF SUSPENDED MAGNETIC PARTICLES THROUGH POROUS MEDIUM

Navdeep Singh Sidhu
Department of Mathematics, GHG Khalsa College Gurusar Sadhar,
Ludhiana, India
e-mail: nvdpsidhu@yahoo.com

ABSTRACT
The transport of vorticity in Rivlin-Ericksen viscoelastic fluid in the presence of suspended magnetic particles in porous medium is considered. Equations governing the transport of vorticity in Rivlin-Ericksen viscoelastic fluid in the presence of suspended magnetic particles in porous medium are obtained from the equations of magnetic fluid flow proposed by Wagh and Jawandhia (1996). A two-dimensional case is also studied.
Key words: Rivlin-Ericksen viscoelastic fluid, suspended particles, vorticity, porous medium.

INTRODUCTION

Magnetic fluids are those fluids in which magnetic particles are suspended in a liquid carrier. Thus, it is a two-phase system, consisting of solid and liquid phases. We shall suppose that the liquid phase is non-magnetic in nature and magnetic force acts only on the magnetic particles. Thus, the magnetic force changes the velocity of the magnetic particles. Consequently, the dragging force acting on the carrier liquid is changed and thus the flow of carrier liquid is also influenced by the magnetic force. Due to the relative velocity between the solid and liquid particles, the net effect of the particles suspended in the fluid is an extra dragging force acting on the system. In recent years, there has been considerable interest in the study of magnetic fluids. Saffman (1962) proposed the equations of the flow of suspension of non-magnetic particles. These equations were modified by Wagh (1991) to describe the flow of magnetic fluid, by including the magnetic body force $\mu_0 M \nabla H$. Wagh and Jawandhia (1996) have studied the transport of vorticity in a magnetic fluid.

With the growing importance of non-Newtonian fluids in modern technology and industries, investigations on such fluids are desirable. Oldroyd (1958) proposed a theoretical model for a class of viscoelastic fluids. Toms and Strawbridge (1953) revealed that a dilute solution of methyl methacrylate in n-butyl acetate agrees well with the theoretical model of Oldroyd fluid. The thermal instability of Maxwellian viscoelastic fluid in the presence of a uniform rotation has been considered by Bhatia and Steiner (1972), where rotation is found to have a destabilizing effect. This is in contrast to the thermal instability of a Newtonian fluid where rotation has a stabilizing effect. The thermal instability of an Oldroydian

viscoelastic fluid acted on by a uniform rotation has been studied by Sharma (1976). There are many viscoelastic fluids that cannot be characterized by Maxwell's or Oldroyd's constitutive relations. The Rivlin-Ericksen viscoelastic fluid is one such fluid. Rivlin and Ericksen (1955) have studied the stress-deformation relaxation for isotropic materials. Garg et al. (1994) have studied the drag on a sphere oscillating in conducting dusty Rivlin-Ericksen elastico-viscous liquid. Thermal instability in Rivlin-Ericksen elastico-viscous fluid in presence of rotation and magnetic field, separately, has been investigated by Sharma and Kumar (1996, 1997a). Sharma and Kumar (1997b) have studied the hydromagnetic stability of two Rivlin-Ericksen elastico-viscous superposed conducting fluids and the analysis has been carried out, for two highly viscous fluids of equal kinematic viscosities and equal kinematic viscoelasticities. It is found that the stability criterion is independent of the effects of viscosity and viscoelasticity and is dependent on the orientation and magnitude of the magnetic field. Kumar and Singh (2006) have studied the stability of two superposed Rivlin-Ericksen viscoelastic fluids in the presence of suspended particles. In another study, Kumar et al. (2007) have studied the hydrodynamic and hydromagnetic stability of two stratified Rivlin-Ericksen elastico-viscous superposed fluids. The medium has been considered to be non-porous in all the above studies.

The flow through porous media is of considerable interest for petroleum engineers and geophysical fluid dynamicists. A great number of applications in geophysics may be found in the books by Phillips (1991), Ingham and Pop (1998), and Nield and Bejan (1999). When the fluid slowly percolates through the pores of a macroscopically homogeneous and isotropic porous medium, the gross effect is represented by the Darcy's law. As a result of this macroscopic law, the usual viscous term in the equations of fluid motion is replaced by the resistance term $-\dfrac{1}{k_1}\left(\mu + \mu'\dfrac{\partial}{\partial t}\right)\vec{q}$, where μ and μ' are the viscosity and viscoelasticity of the Rivlin-Ericksen fluid, k_1 is the medium permeability and $\vec{q}$ is the Darcian (filter) velocity of the fluid. Lapwood (1948) has studied the stability of convective flow in hydromagnetics in a porous medium using Rayleigh's procedure. The Rayleigh instability of a thermal boundary layer in flow through porous medium has been considered by Wooding (1960). The stability of superposed Rivlin-Ericksen elastico-viscous fluids permeated with suspended particles in a porous medium has been considered by Kumar (2000). In another study, Kumar et al. (2004) have considered the instability of two rotating viscoelastic (Rivlin-Ericksen) superposed fluids with suspended particles in porous medium. Kumar et al. (2005) have considered the MHD instability of rotating superposed Rivlin-Ericksen viscoelastic fluids through porous medium.

Keeping in mind the importance of non-Newtonian fluids in modern technology and industries and owing to the importance of porous medium in chemical engineering and geophysics, the present paper attempts to study the transport of vorticity in magnetic Rivlin-Ericksen viscoelastic fluid-particle mixtures in porous medium by using the equations proposed by Wagh and Jawandhia (1996).

BASIC ASSUMPTIONS AND MAGNETIC BODY FORCE

Particles of magnetic material are much larger than the size of the molecules of carrier liquid. Accordingly considering the limit of a microscopic volume element in which a fluid can be assumed to be a continuous medium and the magnetic particles must be treated as discrete entities. Now, if one considers a cell of magnetic fluid containing a larger number of magnetic particles then one must consider the micro rotation of the cell in addition to its translations as a point mass. Thus, one has to assign average velocity $\vec{q}_d$ and the average angular velocity $\vec{\omega}$ of the cell. But, here as an approximation, we neglect the effect of micro rotation. We shall also make the following assumptions:

(i) Most of the ferrofluids are relatively poor conductors and hence free current density $\vec{J}$ is negligible and hence $\vec{J} \times \vec{B}$ is assumed to be insignificant.

(ii) The magnetic field is assumed to be curl free i.e. $\nabla \times \vec{H} = 0$.

(iii) In many practical situations liquid compressibility is not important. Hence contribution due to magnetic friction can be neglected. The remaining force of magnetic field is referred as magnetization force.

(iv) All time-dependent magnetization effects in the fluid such as hysteresis are assumed to be negligible and the magnetization $\vec{M}$ is assumed to be collinear with $\vec{H}$.

From electromagnetic theory, the force per unit volume in MKS units on a piece of magnetized material of magnetization $\vec{M}$ (i.e. dipole moment per unit volume) in the field of magnetic intensity $\vec{H}$ is $\mu_0 (\vec{M}.\nabla)\vec{H}$, where μ_0 is the free space permeability.

Here we note that $\nabla.\vec{a} = \dfrac{\partial a_1}{\partial x} + \dfrac{\partial a_2}{\partial y} + \dfrac{\partial a_3}{\partial z}$ and $\vec{a}.\nabla = a_1 \dfrac{\partial}{\partial x} + a_2 \dfrac{\partial}{\partial y} + a_3 \dfrac{\partial}{\partial z}$, where $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$.

Using assumption (iv), we get

$$\mu_0 (\vec{M}.\nabla)\vec{H} = \frac{\mu_0 M}{H}(\vec{H}.\nabla)\vec{H}, \text{ where } M = \left|\vec{M}\right| \text{ and } H = \left|\vec{H}\right|. \qquad \text{...(1)}$$

But $(\vec{H}.\nabla)\vec{H} = \dfrac{1}{2}\nabla(\vec{H}.\vec{H}) - \vec{H} \times (\nabla \times \vec{H}) = \dfrac{1}{2}\nabla(\vec{H}.\vec{H})$ [by assumption (ii)].   ...(2)

Hence $\mu_0 (\vec{M}.\nabla)\vec{H} = \left(\dfrac{\mu_0 M}{H}\right)\dfrac{1}{2}\nabla(\vec{H}.\vec{H}) = \mu_0 M \nabla H$.

Thus the magnetic body force assumes the form [Rosensweig (1997)]

$$\vec{f}_m = \mu_0 M \nabla H. \qquad \text{...(3)}$$

DERIVATION OF EQUATIONS GOVERNING TRANSPORT OF VORTICITY IN MAGNETIC VISCOELASTIC RIVLIN-ERICKSEN FLUID

Wagh (1991) modified the Saffman's equations for flow of suspension to describe the flow of magnetic fluid by including the body force $\mu_0 M \nabla H$ acting on the suspended magnetic

particles. Now the equations expressing the flow of suspended magnetic particles and the flow of viscoelastic Rivlin-Ericksen fluid in which magnetic particles are suspended in porous medium are

$$\frac{mN}{\varepsilon}\left[\frac{\partial \vec{q}_d}{\partial t}+\frac{1}{\varepsilon}(\vec{q}_d.\nabla)\vec{q}_d\right]=mN\vec{g}+\mu_0 M\nabla H+\frac{KN}{\varepsilon}(\vec{q}-\vec{q}_d),\qquad ...(4)$$

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \vec{q}}{\partial t}+\frac{1}{\varepsilon}(\vec{q}.\nabla)\vec{q}\right]=-\nabla P+\rho\vec{g}-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\vec{q}+\frac{KN}{\varepsilon}(\vec{q}_d-\vec{q}),\qquad ...(5)$$

where mN is the mass of particles per unit volume, $K=6\pi\mu\eta$, where η is the particle radius, is the Stokes' drag coeffiecient, N is the number density, ε is the medium porosity and is defined as $\varepsilon=\dfrac{volume\ of\ the\ voids}{total\ volume}$, $(0<\varepsilon<1)$.

For very fluffy foam materials, ε is nearly one and in bed of packed spheres in the range 0.25-0.50.

In the equations of motion for the fluid, the presence of suspended particles adds an extra force term, proportional to the velocity difference between suspended particles and fluid. Since the force exerted by the fluid on the suspended particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the suspended particles.
Making use of the Lagrange's vector identities

$$(\vec{q}_d.\nabla)\vec{q}_d=\frac{1}{2}\nabla q_d^2-\vec{q}_d\times\vec{\Omega}, (\vec{q}.\nabla)\vec{q}=\frac{1}{2}\nabla q^2-\vec{q}\times\vec{\Omega}_1 ,\qquad ...(6)$$

Equations (4) and (5) become

$$\frac{mN}{\varepsilon}\left[\frac{\partial \vec{q}_d}{\partial t}-\frac{1}{\varepsilon}(\vec{q}_d\times\vec{\Omega})\right]=-\nabla mNgz-\frac{1}{2\varepsilon^2}mN\nabla q_d^2+\mu_0 M\nabla H+\frac{KN}{\varepsilon}(\vec{q}-\vec{q}_d),\qquad ...(7)$$

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \vec{q}}{\partial t}-\frac{1}{\varepsilon}(\vec{q}\times\vec{\Omega}_1)\right]=-\nabla P-\nabla\rho gz-\frac{1}{2\varepsilon^2}\rho\nabla q^2-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\vec{q}+\frac{KN}{\varepsilon}(\vec{q}_d-\vec{q}),\qquad ...(8)$$

where $\vec{\Omega}=\nabla\times\vec{q}_d$ and $\vec{\Omega}_1=\nabla\times\vec{q}$ are solid vorticity and fluid vorticity.

Taking the curl of these equations and keeping that the curl of a gradient is identically zero, we have

$$\frac{mN}{\varepsilon}\left[\frac{\partial \vec{\Omega}}{\partial t}-\frac{1}{\varepsilon}(\nabla\times\vec{q}_d\times\vec{\Omega})\right]=\mu_0\nabla\times M\nabla H+\frac{KN}{\varepsilon}(\vec{\Omega}_1-\vec{\Omega}),\qquad ...(9)$$

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \vec{\Omega}_1}{\partial t}-\frac{1}{\varepsilon}(\nabla\times\vec{q}\times\vec{\Omega}_1)\right]=-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\vec{\Omega}_1+\frac{KN}{\varepsilon}(\vec{\Omega}-\vec{\Omega}_1).\qquad ...(10)$$

By making use of the vector identities

$$\nabla\times(\vec{q}_d\times\vec{\Omega})=(\vec{\Omega}.\nabla)\vec{q}_d-(\vec{q}_d.\nabla)\vec{\Omega}+\vec{q}_d\nabla.\vec{\Omega}-\vec{\Omega}\nabla.\vec{q}_d=(\vec{\Omega}.\nabla)\vec{q}_d-(\vec{q}_d.\nabla)\vec{\Omega} ,\qquad ...(11)$$

$$\nabla\times(\vec{q}\times\vec{\Omega}_1)=(\vec{\Omega}_1.\nabla)\vec{q}-(\vec{q}.\nabla)\vec{\Omega}_1+\vec{q}\nabla.\vec{\Omega}_1-\vec{\Omega}_1\nabla.\vec{q}=(\vec{\Omega}_1.\nabla)\vec{q}-(\vec{q}.\nabla)\vec{\Omega}_1 ,\qquad ...(12)$$

Eqs. (9) and (10) become

$$\frac{mN}{\varepsilon}\frac{D\vec{\Omega}}{Dt}=\mu_0\nabla\times M\nabla H+\frac{mN}{\varepsilon^2}(\vec{\Omega}.\nabla)\vec{q}_d+\frac{KN}{\varepsilon}(\vec{\Omega}_1-\vec{\Omega}),\qquad ...(13)$$

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$$\frac{D\vec{\Omega}_1}{Dt} = -\frac{\varepsilon}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{\Omega}_1 + \frac{1}{\varepsilon}\left(\vec{\Omega}_1 . \nabla\right)\vec{q} + \frac{KN}{\rho}\left(\vec{\Omega} - \vec{\Omega}_1\right), \quad\quad\quad …(14)$$

where ν and ν' are kinematic viscosity and kinematic viscoelasticity, respectively and

$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{1}{\varepsilon}\left(\vec{q}_d . \nabla\right)$ is convective derivative.

In Eq. (13),

$$\nabla \times \left(M\nabla H\right) = \left(\nabla M \times \nabla H\right) + \left(M\nabla \times \nabla H\right). \quad\quad\quad …(15)$$

Since the curl of the gradient is zero, the last term in Eq. (15) is zero. Also since $M = M\left(H, T\right)$.

Therefore, $\nabla M = \left(\frac{\partial M}{\partial H}\right)\nabla H + \left(\frac{\partial M}{\partial T}\right)\nabla T. \quad\quad\quad …(16)$

By making use of (16), Equation (15) becomes

$$\nabla \times \left(M\nabla H\right) = \left(\frac{\partial M}{\partial H}\right)\nabla H \times \nabla H + \left(\frac{\partial M}{\partial T}\right)\nabla T \times \nabla H . \quad\quad\quad …(17)$$

The first term on the right hand side of this equation is clearly zero, hence we get

$$\nabla \times \left(M\nabla H\right) = \left(\frac{\partial M}{\partial T}\right)\nabla T \times \nabla H . \quad\quad\quad …(18)$$

Putting this in Eq. (13), we get

$$\frac{mN}{\varepsilon}\frac{D\vec{\Omega}}{Dt} = \mu_0\left(\frac{\partial M}{\partial T}\right)\nabla T \times \nabla H + \frac{mN}{\varepsilon^2}\left(\vec{\Omega} . \nabla\right)\vec{q}_d + \frac{KN}{\varepsilon}\left(\vec{\Omega}_1 - \vec{\Omega}\right). \quad\quad\quad …(19)$$

Here (14) and (19) are the equations governing the transport of vorticity in magnetic viscoelastic Rivlin-Ericksen fluid-particle mixtures in porous medium.

In Equation (19), the first term $\mu_0\left(\frac{\partial M}{\partial T}\right)\nabla T \times \nabla H$ describes the production of vorticity due to thermo-kinetic processes. The last term $\frac{KN}{\varepsilon}\left(\vec{\Omega}_1 - \vec{\Omega}\right)$ gives the change in solid vorticity on account of exchange of vorticity between the liquid and solid in porous medium.

From Equations (14) and (19), it follows that the transport of solid vorticity $\vec{\Omega}$ is coupled with the transport of fluid vorticity $\vec{\Omega}_1$ in porous medium.

From Equation (19), we see that if solid vorticity $\vec{\Omega}$ is zero, then the fluid vorticity $\vec{\Omega}_1$ is not zero, but it is given by

$$\vec{\Omega}_1 = -\frac{\varepsilon\mu_0}{KN}\left(\frac{\partial M}{\partial T}\right)\nabla T \times \nabla H. \quad\quad\quad …(20)$$

This implies that due to thermo-kinetic process, fluid vorticity may exist in the absence of solid vorticity in porous medium. Equation (20) also shows that fluid vorticity decreases in the presence of porosity. In the absence of porous medium $\left(\varepsilon = 1\right)$

$$\vec{\Omega}_1 = -\frac{\mu_0}{KN}\left(\frac{\partial M}{\partial T}\right)\nabla T \times \nabla H. \quad\quad\quad …(21)$$

This is in conformity with Wagh and Jawandhia (1996) result.

From Eq. (14), we find that if $\vec{\Omega}_1$ is zero, then $\vec{\Omega}$ is also zero. This implies that **when fluid vorticity is zero, then solid vorticity is necessarily zero.**

In the absence of suspended magnetic particles, N is zero and magnetization M is also zero, so Equation (19) is identically satisfied and Eq. (14) reduces to

$$\frac{D\vec{\Omega}_1}{Dt} = -\frac{\varepsilon}{k_1}\left(\nu + \nu'\frac{\partial}{\partial t}\right)\vec{\Omega}_1 + \frac{1}{\varepsilon}\left(\vec{\Omega}_1.\nabla\right)\vec{q}. \qquad \text{...(22)}$$

This equation is vorticity transport equation in porous medium. The last term on the right hand side of Equation (22) represents the rate at which $\vec{\Omega}_1$ varies for a given particle, when the vortex lines move with the fluid, the strengths of the vortices remaining constant. The first term represents the rate of dissipation of vorticity through friction (resistance) and rate of change of vorticity due to fluid viscoelasticity.

<h3 style="text-align:center">TWO-DIMENSIONAL CASE</h3>

Here we consider the two-dimensional case:

Let $\vec{q}_d = q_{d_x}(x,y)\hat{i} + q_{d_y}(x,y)\hat{j}$, $\vec{q} = q_x(x,y)\hat{i} + q_y(x,y)\hat{j}$, $\qquad (23)$

where components q_{d_x}, q_{d_y} and q_x, q_y are functions of x, y and t, then

$$\vec{\Omega} = \Omega_z\hat{k} , \quad \vec{\Omega}_1 = \Omega_{1z}\hat{k} . \qquad (24)$$

In two-dimensional case, Equation (19) becomes

$$\frac{D\Omega_z}{Dt} = \frac{\mu_0\varepsilon}{mN}\left(\frac{\partial M}{\partial T}\right)\left(\frac{\partial T}{\partial x}\frac{\partial H}{\partial y} - \frac{\partial H}{\partial x}\frac{\partial T}{\partial y}\right) + \frac{K}{m}\left(\Omega_{1z} - \Omega_z\right). \qquad (25)$$

Similarly, Eq. (14) becomes

$$\frac{D\Omega_{1z}}{Dt} = -\frac{\varepsilon\nu}{k_1}\Omega_{1z} - \frac{\varepsilon\nu'}{k_1}\frac{\partial}{\partial t}\Omega_{1z} + \frac{KN}{\rho}\left(\Omega_z - \Omega_{1z}\right), \qquad (26)$$

since it can be easily verified that

$$\left(\vec{\Omega}.\nabla\right)\vec{q}_d = 0 \text{ and } \left(\vec{\Omega}_1.\nabla\right)\vec{q} = 0. \qquad (27)$$

The first term on the right hand side of Equation (26) is the change of fluid vorticity due to internal friction (resistance), the second term is the rate of change of fluid vorticity due to fluid viscoelasticity and the third term is change in fluid vorticity on account of exchange of vorticity between solid and liquid. Equation (26) does not involve explicitly the term representing change of vorticity due to magnetic field gradient and/or temperature gradient. But Eq. (25) shows that solid vorticity Ω_z depends on these factors. Hence, it follows that **fluid vorticity is indirectly influenced by the temperature and the magnetic field gradient.**

In the absence of magnetic particles, N is zero and magnetization M is also zero, so Equation (25) is identically satisfied and Equation (26) reduces to classical equation of transport of vorticity for fluid in porous medium. If instead of magnetic field we consider a suspension of non-magnetic particles, then the corresponding equation for the transport of vorticity may be obtained by putting M equal to zero in the equations governing the transport of vorticity in magnetic fluids. If magnetization M of the magnetic particles is independent of

temperature, then the first term of Equations (19) and (25) vanish and so the equations governing the transport of vorticity in magnetic fluid in porous medium are same as those which govern the transport of vorticity in non-magnetic suspensions in porous medium.

If the temperature gradient ∇T vanishes or if the magnetic field gradient ∇H vanishes or if ∇T is parallel to ∇H, then also the first term of Equations (19) and (25) vanish. Thus, we see that in this case also the transport of vorticity in magnetic fluid in porous medium is same as transport of vorticity in non-magnetic suspension in porous medium.

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