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THERMODYNAMIC ANALYSIS OF IMPERFECT REGENERATION STIRLING CYCLE CONSIDERING THE EFFECT OF FRICTION AND DEAD VOLUMES

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Abstract

A model of irreversible Stirling cycle is represented in this paper. The finite time thermodynamic analysis is taken into account in addition with general irreversibility's such as friction and several irreversibility's due to dead volume of hot space, cold space and regenerator are studied in this paper. The analytical formula of power output and efficiency are derived, including effects of friction and dead volumes. Also, the effect of friction, dead volume and regenerator effectiveness are studied by performing the numerical simulation. The result from this study shows that the cycle net work is decreasing with increasing dead volume. Power output and cycle efficiency will depend upon the effectiveness of heat exchangers and dead volume.
Keywords: Stirling cycle, dead volume, frictional loss, irreversible regeneration.

I. INTRODUCTION

The Stirling engine was invented in 1816 by Robert Stirling in Scotland, some 80 years before the invention of a diesel engine. The Stirling engine is a simple type of external combustion engine that uses the compressible fluid as a working fluid with great potential. Being an external combustion engine, it utilizes a wide variety of energy resources: solar, atomic, geothermal, industrial waste heat and marsh-gas.

Our studies and recent investigations by others have found that irreversibilities in the thermodynamic cycle have significant importance in predicting the performance of Stirling engines.

The development of finite-time thermodynamics, a new discipline in modern thermodynamics, provides a powerful tool for performance analysis of practical engineering cycles. Several authors have studied the finite-time thermodynamic performance of the Stirling engine.

Kaushik and Kumar shows an investigation of a finite-time thermodynamic analysis of an

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endoreversible Stirling engine to maximize the power output and the corresponding thermal efficiency of an endoreversible Stirling heat engine with internal heat loss in the regenerator and for the finite heat capacity of the external reservoirs.

Significant results have been obtained by Popescu et al, who studied the performance of the engine by finite-time thermodynamic optimization of an endo-/exo-irreversible Stirling engine and showed that the most important reduction in performance is due to the non-adiabatic regenerator.

Angulo-Brown et al. and Chen et al. optimized the power of the Otto engine and Diesel with friction loss during finite times.

Dead volume is defined as the total void volume in a Stirling engine. In general, the dead volume is referred to as the volume of working fluid contained in the total dead space in the engine, including the regenerator and transfer port. In practical, the Stirling engine must have some unavoidable dead volume.

Schmidt also analyzed the Stirling engine including dead volume. However, ideal regeneration is assumed in the Schmidt analysis. The correct working fluid temperature in the regenerator is important for Stirling engines with large dead volumes. The regenerator effective temperature in another way is the arithmetic mean and the log mean temperatures of working fluid at the outlet and inlet of the regenerator.

This paper shows the thermodynamic analysis of regenerative stirling cycle considering Dead volumes, friction, and imperfect regeneration,

Here the analytical formula of Net Power output and efficiency is derived including the effect of Dead volume and other irreversibilities.

II. SYSTEM DESCRIPTION

Thermal energy must be transferred into and out of the engine via heat exchangers at the hot and cold ends because the heat source of the Stirling engine is external to the working fluid. Therefore there must be a temperature difference between the working fluid and the heat source when the engine receives thermal energy and between the working fluid and the heat sink when it rejects thermal energy.

The engine operates between a constant temperature heat source with temperature TH and a heat sink with temperature TL.

The processes undergoing in the Stirling cycles are:

Process 1-2: Constant Compression

(The Heat is rejected to external sink)

Process 2-3: Constant volume regeneration

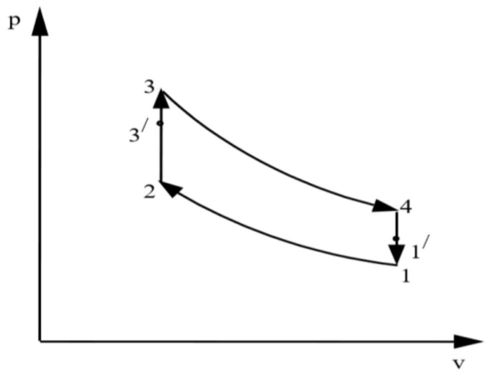
(Internal heat transfer from the regenerator to working fluid)

Process 3-4: Constant expansion

(Heat addition from an external source)

Process 4-1: Constant volume regeneration

(Internal heat transfer from working fluid to regenerator)



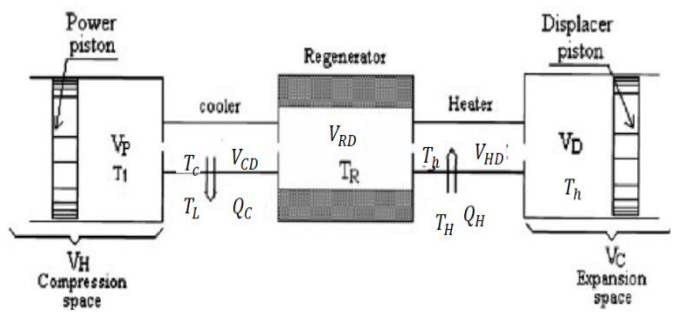


Fig.1.p-v diagram Fig.2. Illustration of the

Fig.1 shows the p-v diagram of regenerative Stirling cycle

For an ideal regeneration, the total heat rejected during process 4-1 is absorbed by a perfect regenerator and released to the working fluid during process 2-3 since in the ideal regeneration an infinite heat-transfer area or an infinite regeneration time is needed.

For an imperfect regenerator, the working fluid temperature at the regenerator outlet and inlet will be T_3 and T_1, respectively.

III. FINITE TIME THERMODYNAMICS ANALYSIS

When the effect of finite heat transfer on the performance of thermodynamic cycle is considered, the amount of heat transferred from Source and Sink to working fluid is written as: [5-6]

$$Q_{in} = k(T_H - T_h)t_h \tag{1}$$

$$Q_{out} = k(T_c - T_L)t_c \tag{2}$$

Where, t_h and t_c are heat addition and heat rejection times for the system. And k is the heat conductance between engine and reservoir.

Also, Heat transfer in the regenerative process is given by:

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$$Q_R = mC_v \in_R (T_h - T_c) \tag{3}$$

Heat loss in the two regenerative processes in the regenerator is given by:

$$\Delta Q_R = mC_v(1 - \in_R)(T_h - T_c) \tag{4}$$

For perfect regeneration,

$$\in_R = 1$$

So,

$$\Delta Q_R = 0$$

Also, time for regenerative heat transfer processes should be considered in the FTT analysis of Stirling engine.

It is assumed that the temperature of the engine working fluid is varying with time in the regenerative process is given by,

$$\frac{dT}{dt} = \pm\alpha$$

Where, α is the proportionality constant which is independent of the temperature but dependent on the property of the regenerative material.

$$t_R = 2\alpha(T_h - T_c) \tag{5}$$

Thus the Total cycle time is given by;

$$t = t_h + t_R + t_c \tag{6}$$

IV. DEAD VOLUME ANALYSIS

Assume that the hot-space, cold space and regenerator dead volumes (in m3), are respectively.V_{HD}, V_{CD} and V_{RD}

Then, the total dead volume is given by:

$$V_{TD} = V_{HD} + V_{CD} + V_{RD} \tag{7}$$

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Dead volume Ratio:

$$K_{HD} = {}^{V_{HD}}/_{V_{TD}} \quad - Hot\ space\ dead\ vol.\ ratio$$

$$K_{CD} = {}^{V_{CD}}/_{V_{TD}} \quad - cold\ space\ dead\ vol.\ ratio$$

$$K_{RD} = {}^{V_{RD}}/_{V_{TD}} \quad - regenerator\ dead\ vol.\ ratio$$

$$K_{TD} = {}^{V_{TD}}/_{V_{T}} \quad - {}^{total\ dead\ vol.}/_{total\ vol.}$$

$$V_{TD} = K_{TD} * V_T = K_{TD}(V_{TD} + V_D + V_P) \tag{8}$$

Where, V_D *and* V_P are the displacer and power piston swept volumes (in m3) respectively.

4.1. Equation of state

$$PV = mRT \tag{9}$$

For isothermal compression;

$$P\left(\frac{V}{T}\right) = mR$$

$$P = \frac{mR}{\left(\frac{V_H}{T_h}+\frac{V_{HD}}{T_h}+\frac{V_{RD}}{T_R}+\frac{V_{CD}}{T_c}+\frac{V_C}{T_c}\right)} = \frac{mR}{\left(\frac{V_H}{T_h}+K+\frac{V_C}{T_c}\right)} \tag{10}$$

Where,

$$K = \left(\frac{V_{HD}}{T_h} + \frac{V_{RD}}{T_R} + \frac{V_{CD}}{T_c}\right) \tag{11}$$

By substituting dead vol. ratios;

$$K = \left(\frac{K_{HD}}{T_h} + \frac{K_{RD}}{T_R} + \frac{K_{CD}}{T_c}\right) V_{TD} \tag{12}$$

Where, $T_R = \frac{T_h + T_c}{2}$

(By using the arithmetic mean the regenerator effective temperature)

For Expansion process, The cold-space working fluid swept volume, V_C, is 0 throughout this process.
Pressure for expansion process is given by:

$$P_e = \frac{mRT_h}{KT_h + V_H} \quad , \quad (V_C = 0) \tag{13}$$

Similarly, for Compression process;
The hot-space working fluid swept volume, V_H is 0 throughout this process.
Pressure for compression process is given by:

$$P_c = \frac{mRT_c}{KT_c + V_C} \quad , \quad (V_H = 0) \tag{14}$$

Where, m is the total working fluid mass contained in the engine (in kg) and R(kJ/kg K) is the universal gas constant.

4.2 Constant Compression

Heat released, [12-14]
In the isothermal compression process, the cold-side working fluid is compressed from V_{C1} to V_{C2}.

Using eq. (14)

$$Q_c = W_c = \int_{V_{C1}}^{V_{C2}} P_c dV_C = mRT_c \int_{V_{C1}}^{V_{C2}} \frac{dV_C}{V_C + KT_c} \tag{15}$$

$$V_{C1} = V_P + V_D$$

$V_{C2} = V_D$
$V_H = 0$

$$Q_c = W_c = mRT_c \ln\left[\frac{V_D + KT_c}{V_P + V_D + KT_c}\right] \tag{16}$$

Let, $\left[\frac{V_P + V_D + KT_c}{V_D + KT_c}\right] = \lambda_1$ (17)

$$Q_c = W_c = mRT_c \ln\left(\frac{1}{\lambda_1}\right) \tag{18}$$

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4.3 Constant Expansion

Heat added;

In the isothermal expansion process, the hot-side working fluid volume changes from V_{H3} to V_{H4}.

Using eq. (13)

$$Q_h = W_h = \int_{V_{H3}}^{V_{H4}} P_e dV_H = mRT_h \ln\left[\frac{V_P+V_D+KT_h}{V_D+KT_h}\right] \quad (19)$$

$V_{H4} = V_P + V_D$

$V_{H3} = V_D$

Let, $\left[\frac{V_P+V_D+KT_h}{V_D+KT_h}\right] = \lambda_2$ (20)

$$Q_h = W_h = mRT_h \ln(\lambda_2) \quad (21)$$

4.4 Constant Volume heat transfer process

The constant volume regeneration process is taking place in the regenerator,

$\in_R -$ $Effectiveness\ of\ regenerator$

Heat transferred by regenerator;

$$Q_R = mC_v \in_R (T_h - T_c) \quad (22)$$

Heat loss by regenerator;

$$\Delta Q_R = mC_v(1 - \in_R)(T_h - T_c) \quad (23)$$

Now,

Net heat absorbed,

$$Q_{in} = Q_H = Q_h + \Delta Q_R \quad (24)$$

Net heat released,

$$Q_{out} = Q_L = Q_c + \Delta Q_R \quad (25)$$

4.5 Net Work

Net Work Output,

$$W_{Net} = Q_H - Q_L = Q_h - Q_c$$

For second law of TD,

$$\oint \frac{dQ}{T} = \frac{Q_h}{T_h} - \frac{Q_c}{T_c} \leq 0$$

$$Q_h = Q_c\left(\frac{T_h}{T_c}\right) \quad (26)$$

By using eqn. (26)

$$W_{Net} = Q_c\left(\frac{T_h}{T_c}\right) - Q_c = Q_c\left(\frac{T_h}{T_c} - 1\right)$$

By using eqn. (16), (17) and (18)

$$W_{Net} = -\left(\frac{T_h}{T_c} - 1\right) mRT_c \ln\left[\frac{V_D + KT_c}{V_P + V_D + KT_c}\right] \quad (27)$$

$$W_{Net} = (T_h - T_c) mRln(\lambda_1) \quad (28)$$

In the case of zero dead volume and reversible cycle, the net work is

$$W_{Net} = mR(T_h - T_c) \ln\left[\frac{V_1}{V_2}\right] \quad (29)$$

V. POWER LOSS BY FRICTION

Taking into account the friction losses of the piston as recommended by Angulo-Brown et al.[7-8]

Friction force is a linear function of velocity.

Friction power,

i.e, $Friction\ force \propto v$

$$F.F = -\mu v = -\mu\left(\frac{dx}{dt}\right) \quad (30)$$

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Frictional power,

$$P_f = -\left(\frac{dW_f}{dt}\right)$$

$$P_f = -\mu\left(\frac{dx}{dt}\right)^2 = -\mu v^2 \tag{31}$$

Where,

$$v = v_{mean} = \frac{x - x_2}{\Delta t_{12}} = \frac{x_2\left(\frac{V_1}{V_2} - 1\right)}{\Delta t_{12}}$$

Let,

$$b = -\frac{\mu x^2}{\Delta t_{12}} \tag{32}$$

$$P_f = -b\left(\frac{V_1}{V_2} - 1\right)^2 \tag{33}$$

VI. POWER OUTPUT

The Power output is given by:

$$P_{out} = \frac{W_{Net}}{t}$$

Using above equations ,

we get Power output as:

$$P_{out} = \left(\frac{Q_h - Q_c}{t_h + t_R + t_c}\right) \tag{34}$$

$$P_{out} = \frac{(T_h - T_c)mR\ln\left[\frac{V_P + V_D + KT_c}{V_D + KT_c}\right]}{t_h + t_R + t_c} \tag{35}$$

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$$P_{out} = \frac{(T_h - T_c) mR \ln\left[\frac{V_P + V_D + KT_c}{V_D + KT_c}\right]}{\frac{Q_{in}}{k(T_H - T_h)} + \frac{Q_{out}}{k(T_c - T_L)} + 2\alpha (T_h - T_c)} \tag{36}$$

From eq. (1), (2), (5), (17), (18), (20) and (21),

We get,

$$P_{out} = \frac{(T_h - T_c) mR \ln[\lambda_1]}{\frac{mRT_h \ln(\lambda_2) + mC_v(1 - \epsilon_R)(T_h - T_c)}{k(T_H - T_h)} + \frac{mRT_c \ln\left(\frac{1}{\lambda_1}\right) + mC_v(1 - \epsilon_R)(T_h - T_c)}{k(T_c - T_L)} + 2\alpha(T_h - T_c)} \tag{37}$$

$$P_{out} = \frac{(T_h - T_c) k \ln[\lambda_1]}{\frac{T_h \ln(\lambda_2) + (\frac{1}{\gamma - 1})(1 - \epsilon_R)(T_h - T_c)}{(T_H - T_h)} + \frac{T_c \ln(\frac{1}{\lambda_1}) + (\frac{1}{\gamma - 1})(1 - \epsilon_R)(T_h - T_c)}{(T_c - T_L)} + \frac{2\alpha k}{mR}(T_h - T_c)} \tag{38}$$

If, $\epsilon_R = 1$

$$P_{out} = \frac{(T_h - T_c) k \ln[\lambda_1]}{\frac{T_h \ln(\lambda_2) +}{(T_H - T_h)} + \frac{T_c \ln(\frac{1}{\lambda_1}) +}{k(T_c - T_L)} + \alpha_1 (T_h - T_c)} \tag{39}$$

Where,

$$\alpha_1 = \frac{2\alpha k}{mR} \tag{40}$$

VII. ACTUAL POWER OUTPUT

The actual power output considering frictional power loss
Is given by:

$P_{act} = P_{out} - P_f$

By using eq. (33) and (38)

We have the actual power output as:

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$$P_{act} = \frac{(T_h - T_c) k \ln[\lambda_1]}{\frac{T_h \ln(\lambda_2) + \left(\frac{1}{\gamma - 1}\right)(1 - \epsilon_R)(T_h - T_c)}{(T_H - T_h)} + \frac{T_c \ln\left(\frac{1}{\lambda_1}\right) + \left(\frac{1}{\gamma - 1}\right)(1 - \epsilon_R)(T_h - T_c)}{(T_c - T_L)} + \alpha_1 (T_h - T_c)} - b\left(\frac{V_1}{V_2} - 1\right)^2 \quad (41)$$

VIII. EFFICIENCY

And the thermal efficiency is given by:

$$\eta = \frac{W_{Net}}{Q_H} = \frac{W_{Net}}{Q_h + \Delta Q_R}$$

from eq. (18), (20), (21), (27) and (28),
we get the thermal efficiency, which is given by:

$$\eta = \frac{(T_h - T_c) mR \ln\left[\frac{V_P + V_D + KT_c}{V_D + KT_c}\right]}{mRT_h \ln\left[\frac{V_P + V_D + KT_h}{V_D + KT_h}\right] + mC_v (1 - \epsilon_R)(T_h - T_c)} \quad (42)$$

$$\eta = \frac{(T_h - T_c) \ln[\lambda_1]}{T_h \ln[\lambda_2] + \frac{1}{(\gamma - 1)}(1 - \epsilon_R)(T_h - T_c)} \quad (43)$$

If $\epsilon_R = 1$
Then,

$$\eta = \left(1 - \frac{T_C}{T_h}\right)\frac{\ln \lambda_1}{\ln \lambda_2} \quad (44)$$

Equation (41) and (43) are the main results of this analysis.

IX. RESULTS

From the above analysis we can see that,
The actual power output is depends upon dead volume, heat conductance between engine and reservoir (k), effectiveness of regenerator (ϵ_R).
The efficiency is depends on Th, Tc, dead volume and (ϵ_R).
Plots of effect of total dead volume and imperfect regeneration are shown below.

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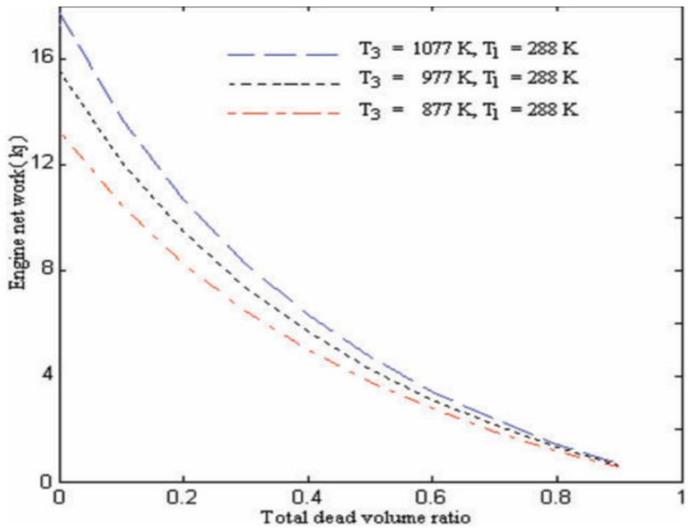

Fig.3. Fig Engine net work against total dead volume ratio

Fig.3. shows the variation of Engine net Work with total dead volume ratio.
It is seen that engine net work decreases as total dead volume ratio increases.

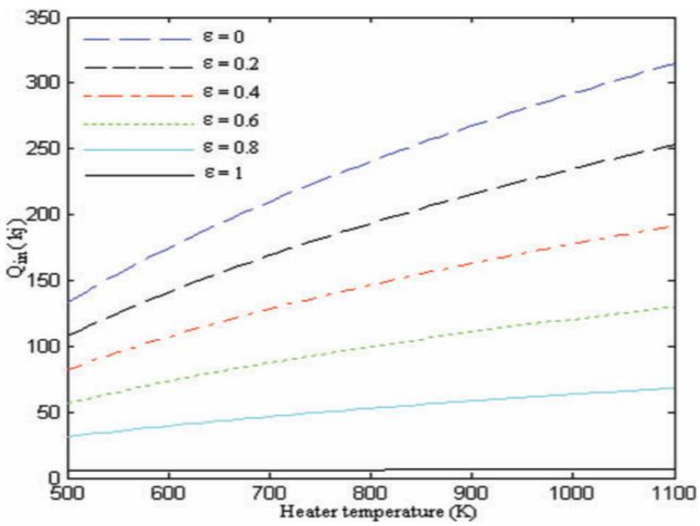

Fig.4. 8 Total heat input against heater temperature at different values of regenerator effectiveness

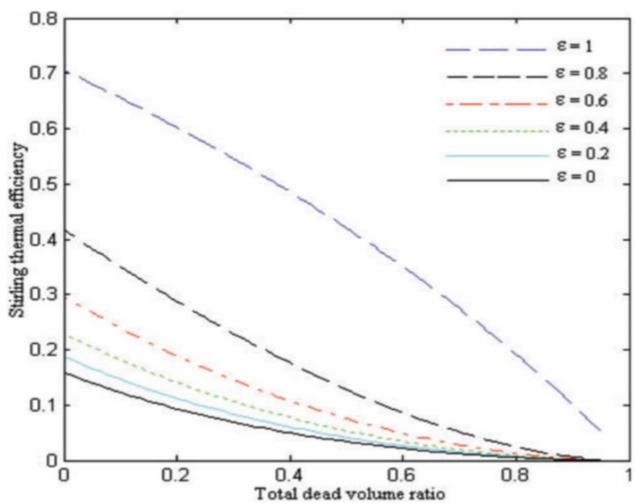

Fig.5. **11** Stirling thermal efficiency against total dead volume ratio at different values of regenerator effectiveness

Fig.4 shows the effect of heater temperature on heat input consiering the effect of imperfect regeneration and
Fig.5. shows the effect of total dead volume ratio on thermal efficiency with regenerative losses.

REFERENCES

[1] Robert Stirling, Patent no. 4081, Stirling air engine and the heat regenerator, 1816.
[2] Curzon FL, Ahlborn B. Efficiency of Carnot heat engine at maximum power output. Am J Phys 1975;43:22–4.[3] Lee WY, Kim SS. Finite time optimization of Rankine heat engine. Energy Conv Mangt 1992;33(1):59–67.
[3] Wu C, Chen L, Sun F. Performance of regenerative Brayton heat engine. Energy 1996;21(2):71–6.
[4] Kaushik SC, Kumar S. Finite time thermodynamic analysis of a nonisentropic regenerative Brayton heat engine.Int J Solar Energy, submitted, 1999
[5] Kaushik SC, Kumar S. Finite time thermodynamic analysis of endoreversible Stirling heat engine with regenerative losses. Energy 2000;25(10):989–1003.
[6] Angulo-Brown F, Fernandez-Betanzos J, Diaz-Pico CA. 1994. Compression ratio of an optimized Otto-cycle model.European Journal of Physics 15:38–42
[7] Angulo-Brown F, Rocha-Martinez JA, Navarrete-Gonzalez TD. 1996. A non-endoreversible Otto cycle model:improving power output and efficiency. Journal of Physics D: Applied Physics 29:80–83.
[8] L. Chen, J. Lin, J. Lou, F. Sun, C. Wu, Friction effect on the characteristic performance of Diesel engines, Internat. J. Energy Res., in press

[9] Feng Wu, Lingen Chen, Chih Wu, Fengrui Sun. Optimum performance of irreversible Stirling engine with imperfect regeneration. J Energy Conserv Manage 1998;39(8):727–32

[10] International Journal of Advance Engineering and Research Development (IJAERD) Volume 2,Issue 2, February -2015, e-ISSN: 2348 - 4470 , print-ISSN:2348-6406

[11] Tlili, Y Timoumi and S B Nasrallah Thermodynamic analysis of the Stirling heat engine with regenerative losses and internal irreversibilities International Journal of Engine Research 2008 9:45

[12] Wu, F., Chen, L., Wu, C., and Sun, F. Optimum performance of irreversible Stirling engine with imperfect regeneration. Energ. Convers. Mgmt, Energy Convers. Mgmt Vol. 39, No. 8, pp. 727-732, 1998

[13] D.G. Thombare , S.K. Verma Technological development in the Stirling cycle engines

[14] Bancha Kongtragool, Somchai Wongwises, Thermodynamic analysis of a Stirling engine including dead volumes of hot space, cold space and regenerator. Renewable Energy 31 (2006) 345–359