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A Non Mathematical Exposition of Game Theory

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ABSTRACT

The famous statement of J B S Haldane an ounce of algebra is worth a ton of verbal argument (as quoted by John Maynard Smith) notwithstanding, an attempt is being made to explain game theory non-mathematically. This paper has been written with the intent of educating students in management sciences with the nuances of Game Theory which should be covered as a part of their Quantitative Methods as well as Business Strategy curriculum. It is this author's belief that students going through a semester's course on Quantitative Methods and another on Business Strategy without covering Game Theory, (as is often the case in non-premier institutions), ought to feel that they have been short-changed. This is sad and it is high time that we who call ourselves "teachers" address this pedagogical lacuna. In social science too many students are found to be not so strong in quantitative methods and an understanding of game theory will help enrich their understanding in subjects like sociology and political science. A symbiosis between natural sciences and social sciences can thus be approximated. This is a modest attempt in that direction.

Introduction

Games or 'Strategic Interactions' can be found in all walks of life. Examples of such scenarios are two firms competing for market share, politicians contesting elections, family members vying for share of funds or property, different bidders participating in an auction for wireless spectrum, coal blocks etc. Game theory provides a convenient framework to model and helps us to interpret the behaviour of participants in such strategic interactions. Hence it can be applied to solve a wide variety of problems involving diverse areas such as industrial relations, markets, auctions, online retail, cold war, paying taxes, bargaining, elections, portfolio management, social interactions etc. Game theory could thus be viewed as a study of strategic decision making. More formally, it is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers". (Myerson and later Anand) An alternative term suggested "as a more descriptive name for the discipline" is interactive decision theory. Game theory is mainly used in

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economics, political science, and psychology, as well as logic and biology. The subject first addressed zero-sum games, such that one person's gains exactly equal net losses of the other participant(s). Today, however, game theory applies to a wide range of behavioural relations, and has developed into an umbrella term for the logical side of decision science, to include both human and non-humans, like computers. (Aumann and later Shoemaker)

Mathematical Roots

It can be argued that Game Theory is a branch of applied mathematics that is often used in the context of economics and productivity related sciences. It studies strategic interactions between agents. In strategic games, agents choose strategies which will maximize their return, given the strategies the other agents choose. The essential feature is that it provides a formal modelling approach to social situations in which decision makers interact with other agents. Game theory extends the simpler optimisation approach developed in neoclassical economics. However, there is a common perception that game theory is a mathematical construct and therefore many scholars shy away from it. What this paper sets out to do is to explain game theory in a non-mathematical manner so that its understanding would give it the wide intellectual appeal that it richly deserves.

History of game theory

The first known discussion of game theory is said to have occurred in a letter written by James Waldegrave in 1713. In this letter, Waldegrave provided a mini-max mixed strategy solution to a two-person version of the card game le Her. It was not until the publication of Antoine Augustin Cournot's Researches into the Mathematical Principles of the Theory of Wealth in 1838 that a general game theoretic analysis was pursued. In this work Cournot considered a duopoly and presents a solution that is a restricted version of the Nash equilibrium. In game theory, the Nash equilibrium is a solution concept of a non-cooperative game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only their own strategy. If each player has chosen a strategy and no player can benefit by changing strategies while the other players keep their unchanged. At this point the current set of strategy choices and the corresponding payoffs constitute Nash equilibrium. Stated simply, two girls, Arti and Bharati are in Nash equilibrium if Arti is making the best decision she can, taking into account Bharati's decision, and Bharati is making the best decision he can, taking into account Arti's decision. Likewise, a group of players are in Nash equilibrium if each one is making the best decision that he or she can, taking into account the decisions of the others.

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Economics

What economists call game theory psychologists call the theory of social situations, which is an accurate description of what game theory is about. Although game theory is relevant to parlour games such as poker or bridge, most research in game theory focuses on how groups of people interact. There are two main branches of game theory: cooperative and non-cooperative game theory. Non-cooperative game theory deals largely with how intelligent individuals interact with one another in an effort to achieve their own goals. In addition to game theory, economic theory has three other main branches: decision theory, general equilibrium theory and mechanism design theory. All are closely connected to game theory and can be used somewhat similarly.

Students of economic science usually cover the Cournot solution in their studies in oligopoly pricing at the undergraduate level. Although Cournot's analysis is more general than Waldegrave's version, game theory *per se* did not really exist as a unique field until John von Neumann published a series of papers in 1928. While the French mathematician Borel did some earlier work on games, Von Neumann can rightfully be credited as the inventor of game theory. Von Neumann was a brilliant mathematician whose work was far-reaching from set theory to his calculations that were critical to development of both the Atom and Hydrogen bombs and finally to his work in developing computers. Von Neumann's work in game theory culminated in the 1944 book The Theory of Games and Economic Behaviour by von Neumann and Oskar Morgenstern. This profound work contained the method for finding optimal solutions for two-person zero-sum games. During this time period, work on game theory was primarily focused on cooperative game theory, which analyzes optimal strategies for groups of individuals, presuming that they can enforce agreements between them about proper strategies.

In 1950, the first discussion of the prisoner's dilemma appeared, and an experiment was undertaken on this game at the RAND Corporation. Around this same time, John Nash developed a definition of an "optimum" strategy for multiplayer games where no such optimum was previously defined, known as Nash equilibrium. This equilibrium is sufficiently general, allowing for the analysis of non-cooperative games in addition to cooperative ones.

Game theory experienced a flurry of activity in the 1950s, during which time the concepts of the core, the extensive form game, fictitious play, repeated games, and the Shapley value were developed. In addition, the first applications of Game theory to philosophy and political science occurred during this time. In 1965, Reinhard Selten introduced his solution concept of sub-game perfect equilibria, which further refined the Nash equilibrium (later he would introduce trembling hand perfection as well). In 1967, John Harsanyi developed the concepts of complete information and Bayesian games. Nash, Selten and Harsanyi became Economics Nobel Laureates in 1994 for their contributions to economic game theory.

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Political science

The application of game theory to political science is focused in the overlapping areas of fair division, political economy, public choice, positive political theory, and social choice theory. In each of these areas, researchers have developed game theoretic models in which the players are often voters, states, interest groups, and politicians.

For early examples of game theory applied to political science, see the work of Anthony Downs. In his book *An Economic Theory of Democracy* (1957), he applies a Hoteling firm location model to the political process. In the Downsian model, political candidates commit to ideologies on a one-dimensional policy space. The theorist shows how the political candidates will converge to the ideology preferred by the median voter. The book has set forth a model with precise conditions under which economic theory could be applied to non-market political decision-making. It also suggested areas of empirical research that could be tested to confirm the validity of his conclusions in the model. Much of this off-shoot research eventually became integrated into the Public Choice School. Downs' theory abstains from making normative statements about public policy choices and instead focuses on what is rational, given the relevant incentives, for government to do. For more recent examples, one could see the works of George Tsebelis, Gene M. Grossman and Elhanan Helpman, or David Austen-Smith and Jeffrey S. Banks [See Gustav Ranis and J Vreeland and Stephen Cosack 2006 for a lucid treatment]. For an application of a similar logic using Arrows Impossibility Theorem in the Indian political context see Sadri

After the dissemination of the oldest political part of India “The Congress” in 2014 all sorts of games were being played by the politicians. No level was too low for them to sink to. Religion, caste, regionalism, allegations of money laundering, and other forms of calumny were rife. This debate intensified after the BJP swept the polls in Jharkhand and made its presence unmistakably felt in Jammu and Kashmir. Various scenarios were painted by news broadcasters in the days that followed. That gave impetus to this author to re-examine “game theory”. The application of game theory to political science is not something new and is usually focused in the overlapping areas of fair division, political economy, public choice, war bargaining, positive political theory, and social. In each of these areas, researchers have developed game-theoretic models in which the players are often voters, states, special interest groups, and politicians. Early examples of game theory applied to political science are provided by Anthony Downs. In his book *An Economic Theory of Democracy*, (Downs 1957) he applies the Hoteling firm location model to the political process. In the Downsian model, political candidates commit to ideologies on a one-dimensional policy space. Downs first shows how the political candidates will converge to the ideology preferred by the median voter if voters are fully informed, but then argues that voters choose to remain rationally ignorant which allows for candidate divergence.

A game-theoretic explanation for democratic peace is that public and open debates in Democracies send clear and reliable information regarding their intentions to other states. In contrast, it is difficult to know the intentions of non-democratic leaders, what effect concessions will have, and if promises will be kept. Thus there will be mistrust and unwillingness to make concessions if at least one of the parties in a dispute is a non democracy.

Game theory provides a theoretical description for a variety of observable consequences of changes in governmental policies. For example, in a static world where producers were not themselves decision makers attempting to optimize their own expenditure of resources while assuming risks, response to an increase in tax rates would imply an increase in revenues and vice versa. Game Theory inclusively weights the decision making of all participants and thus explains the contrary results illustrated by the Laffer curve. This is typically a graphical representation of the relationship between possible rates of taxation and the resulting levels of government revenue. It illustrates the concept of taxable income elasticity—i.e., taxable income will change in response to changes in the rate of taxation. It postulates that no tax revenue will be raised at the extreme tax rates of 0% and 100% and that there must be at least one rate where tax revenue would be a non-zero maximum. One potential result of the Laffer curve hypothesis is that increasing tax rates beyond a certain point will be counter-productive for raising further tax revenue. Hence it does not have many takers from the orthodox Keynesian school.

Business studies

The fundamental cornerstone for a wide variety of specializations in management science (e.g. finance, marketing, operations research and productivity studies) have been indubitably found to lie in the economic science. Economists especially post 1970 have used game theory to analyse a wide array of economic phenomena, including auctions, bargaining, duopolies, fair division, oligopolies, social network formation, and voting systems. This research usually focuses on particular sets of strategies known as equilibria in games. These "solution concepts" are usually based on what is required by norms of rationality. The most famous of these is the Nash equilibrium. A set of strategies is a Nash equilibrium if each represents a best response to the other strategies. So, if all the players are playing the strategies in a Nash equilibrium, they have no unilateral incentive to deviate, since their strategy is the best they can do given what others are doing.

The payoffs of the game are generally taken to represent the utility of individual players. Often in modelling situations the payoffs represent money, which presumably corresponds to an individual's utility. This assumption, however, can be faulty.

A prototypical paper on game theory in economics begins by presenting a game that is an abstraction of some particular economic situation. One or more solution concepts are chosen, and the author demonstrates which strategy sets in the presented game are equilibria of the appropriate type. Naturally one might wonder to what use this information should be put. Economists and business professors suggest two primary uses.

Demography and Sociology

In the 1970s, game theory was extensively applied in biology, largely as a result of the work of John Maynard Smith and his evolutionary stable strategy. In addition, the concepts of correlated equilibrium, trembling hand perfection, and common knowledge were introduced and analysed. In 2005, game theorists Thomas Schelling and Robert Aumann followed Nash, Selten and Harsanyi as Nobel Laureates. Schelling worked on dynamic models of segregation of populace in civil society, presenting us with an early example of evolutionary game theory. Aumann contributed more to the equilibrium school, developing an equilibrium coarsening correlated equilibrium and developing extensive analysis of the assumption of common knowledge.

Whereas the field of game theory came into being almost seven decades ago with the 1944 classic work *Theory of Games and Economic Behaviour* by John von Neumann and Oskar Morgenstern, a major centre for the development of game theory remained the RAND Corporation where it helped to define nuclear strategies. Game theory has played, and continues to play a large role in the social sciences, and is now also used in many diverse academic fields. Beginning in the 1970s, game theory has been applied to animal behaviour, including evolutionary theory. Many games, especially the prisoner's dilemma, are used to illustrate ideas in political science and ethics. Game theory has recently drawn attention from computer scientists because of its use in artificial intelligence and cybernetics.

In addition to its academic interest, game theory has received attention in popular culture. A Nobel Prize–winning game theorist, John Nash, was the subject of the 1998 biography by Sylvia Nasar and the 2001 film A Beautiful Mind. Game theory was also a theme in the 1983 film War Games. Several game shows have adopted game theoretic situations, including identifying the Friend or Foe, and to some extent also the Survivor in a simulated situation. Although some game theoretic analyses appear similar to decision theory, game theory studies decisions made in an environment in which players interact. In other words, game theory studies choice of optimal behaviour when costs and benefits of each option depend upon the choices of other individuals.

Decision theory can be viewed as a theory of one person games, or a game of a single player against nature. The focus is on preferences and the formation of beliefs. The most widely used form of decision theory argues that preferences among risky alternatives can be described by the maximization the expected value of a numerical utility function, where utility may depend on a number of things, but in situations of interest to economists often depends on money income. Probability theory is heavily used in order to represent the uncertainty of outcomes, and Baye's Law in Conditional Probability Theory is frequently used to model the way in which new information is used to revise beliefs. Decision theory is often used in the form of decision analysis, which shows how best to acquire information before making a decision.

General equilibrium theory

It can be viewed as a specialized branch of game theory that deals with trade and production, and typically with a relatively large number of individual consumers and producers. It is widely used in the macroeconomic analysis of broad based economic policies such as monetary or tax policy, in finance to analyse stock markets, to study interest and exchange rates and other prices. In recent years, political economy has emerged as a combination of general equilibrium theory and game theory in which the private sector of the economy is modelled by general equilibrium theory, while voting behaviour and the incentive of governments is analyzed using game theory. Issues studied include tax policy, trade policy, and the role of international trade agreements such as the European Union.

Mechanism design theory

It differs from game theory in that game theory takes the rules of the game as given, while mechanism design theory asks about the consequences of different types of rules. Naturally this relies heavily on game theory. Questions addressed by mechanism design theory include the design of compensation and wage agreements that effectively spread risk while maintaining incentives, and the design of auctions to maximize revenue, or achieve other goals. In Game Theory, we economists believe, various games are mathematical objects of different character — in rules, relative payoffs, and attendant mathematical behaviours. Each "game" represents different situations — in the kinds of problems that organisms have to deal with, and the possible e strategies that they might adopt if they are to successfully survive and reproduce. To achieve a better feel for the challenges of these different situations, evolutionary games are often given rather colourful names and "cover stories" which quite effectively describe the general situation in which the particular game places its players. It all helps develop a feel for the mathematics of the game and the problems the players face.

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Volume 1, Issue 11, February 2015

The economist David K Levine has given a rather instructive example in the following manner. One way to describe a game, he claims, is by listing the players (or individuals) participating in the game, and for each player, listing the alternative choices (called actions or strategies) available to that player. In the case of a two-player game, the actions of the first player form the rows, and the actions of the second player the columns, of a matrix. The entries in the matrix are two numbers representing the utility or payoff to the first and second player respectively.

Prisoner's dilemma

A very famous game is the Prisoner's Dilemma game. In this game the two players are partners in a crime who have been captured by the police. Each suspect is placed in a separate cell, and offered the opportunity to confess to the crime. The game can be represented by the following matrix of payoffs

	Not Confess	Confess
Not Confess	5, 5	0, 10
Confess	10, 0	1, 1

It is worthy of note that higher numbers are better (have more utility). If neither suspect confesses, they go free, and split the proceeds of their crime which we represent by 5 units of utility for each suspect. However, if one prisoner confesses and the other does not, the prisoner who confesses testifies against the other in exchange for going free and gets the entire 10 units of utility, while the prisoner who did not confess goes to prison and gets nothing. If both prisoners confess, then both are given a reduced term, but both are convicted, which we represent by giving each 1 unit of utility: better than having the other prisoner to confess, but not as good as going free.

This game has fascinated game theorists for a variety of reasons. First, it is a simple representation of a variety of important situations. For example, instead of confess/not confess we could label the strategies "contribute to the common good" or "behave selfishly." This captures a variety of situations economists describe as public goods problems. An example is the construction of a bridge. It is best for everyone if the bridge is built, but best for each individual if someone else builds the bridge. This is sometimes referred to in economics as an externality. Similarly this game could describe the alternative of two firms competing in the same market, and instead of confess/not confess we could label the strategies "set a high price" and "set a low price." Naturally is is best for both firms if they both set high prices, but best for each individual firm to set a low price while the opposition sets a high price.

A second feature of this game is that it is self-evident how an intelligent individual should behave. No matter what a suspect believes his partner is going to do, is is always best to confess. If the partner in the other cell is not confessing, it is possible to get 10 instead of 5. If the partner in the other cell is confessing, it is possible to get 1 instead of 0. Yet the pursuit of individually sensible behaviour results in each player getting only 1 unit of utility, much less than the 5 units each that they would get if neither confessed. This conflict between the pursuit of individual goals and the common good is at the heart of many game theoretic problems.

A third feature of this game is that it changes in a very significant way if the game is repeated, or if the players will interact with each other again in the future. Suppose for example that after this game is over, and the suspects either are freed or are released from jail they will commit another crime and the game will be played again. In this case in the first period the suspects may reason that they should not confess because if they do not their partner will not confess in the second game. Strictly speaking, this conclusion is not valid, since in the second game both suspects will confess no matter what happened in the first game. However, repetition opens up the possibility of being rewarded or punished in the future for current behaviour, and game theorists have provided a number of theories to explain the obvious intuition that if the game is repeated often enough, the suspects ought to cooperate.

Extensive form

A game tree (also called the extensive form) is a graphical representation of a sequential game. It provides information about the players, payoffs, strategies, and the order of moves. The game tree consists of nodes (or vertices), which are points at which players can take actions, connected by edges, which represent the actions that may be taken at that node. An initial (or root) node represents the first decision to be made. Every set of edges from the first node through the tree eventually arrives at a terminal node, representing an end to the game. Each terminal node is labelled with the payoffs earned by each player if the game ends at that node.

The games studied by game theory are well-defined mathematical objects. A game consists of a set of players, a set of moves (or strategies) available to those players, and a specification of payoffs for each combination of strategies. Most cooperative games are presented in the characteristic function form, while the extensive and the normal forms are used to define non-cooperative games.

The extensive form of Game Theory can be used to formalize games with some important order. Games here are often presented as trees. Here each vertex (or node) represents a point of choice for a player. The player is specified by a number listed by the vertex. The lines out of the vertex represent a possible action for that player. The payoffs are specified at the bottom of the tree.

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Volume 1, Issue 11, February 2015

Let us picture a common game where there are two players. Player 1 moves first and chooses either F or U. Player 2 sees Player 1's move and then chooses A or R. Suppose that Player 1 chooses U and then Player 2 chooses A, then Player 1 gets 8 and Player 2 gets 2.

The extensive form can also capture simultaneous-move games and games with incomplete information. To represent it, either a dotted line connects different vertices to represent them as being part of the same information set (i.e., the players do not know at which point they are), or a closed line is drawn around them.

The normal (or strategic form) game

This is usually represented by a matrix which shows the players, strategies, and payoffs (see the example to the right). More generally it can be represented by any function that associates a payoff for each player with every possible combination of actions. In the accompanying example there are two players; one chooses the row and the other chooses the column. Each player has two strategies, which are specified by the number of rows and the number of columns. The payoffs are provided in the interior. The first number is the payoff received by the row player (Player 1 in our example); the second is the payoff for the column player (Player 2 in our example). Suppose that Player 1 plays Up and that Player 2 plays Left. Then Player 1 gets a payoff of 4, and Player 2 gets 3.

When a game is presented in normal form, it is presumed that each player acts simultaneously or, at least, without knowing the actions of the other. If players have some information about the choices of other players, the game is usually presented in extensive form.

In cooperative games with transferable utility no individual payoffs are given. Instead, the characteristic function determines the payoff of each coalition. The standard assumption is that the empty coalition obtains a payoff of 0.

The origin of this form is to be found the in the seminal book of von Neumann and Morgenstern who, when studying coalitional normal form games, assumed that when a coalition C forms, it plays against the complementary coalition () as if they were playing a 2-player game. The equilibrium payoff of C is characteristic. Now there are different models to derive coalitional values from normal form games, but not all games in characteristic function form can be derived from normal form games.

Formally, a characteristic function form game (also known as a TU-game) is given as a pair (N,v), where N denotes a set of players and is a characteristic function. The characteristic function form has been generalised to games without the assumption of transferable utility. [TU] Unleashed is another great flight simulation game of the Tupolev plane series. As a pilot it is his/her job to transport passengers to seven tropical islands.

The Partition function form of the characteristic function form ignores the possible externalities of coalition formation. In the partition function form the payoff of a coalition depends not only on its members, but also on the way the rest of the players are partitioned (Thrall & Lucas 1963).

Cooperative game and Non-cooperative game: A game is said to be cooperative if the players are able to form binding commitments. For instance the legal system requires the parties in a negotiation to adhere to their promises. In non-cooperative games this is not possible.

Often it is assumed that communication among players is allowed in cooperative games, but not in non-cooperative ones. This classification on two binary criteria has been rejected (Harsanyi 1974).

Of the two types of games, non-cooperative games are able to model situations to the finest details, producing accurate results. Cooperative games focus on the game at large. Considerable efforts have been made to link the two approaches. The so-called Nash-programme has already established many of the cooperative solutions as non-cooperative equilibria.

Hybrid games contain cooperative and non-cooperative elements. For instance coalitions of players are formed in a cooperative game, but these play in a non-cooperative fashion.

A symmetric game is a game where the payoffs for playing a particular strategy depend only on the other strategies employed, not on who is playing them. If the identities of the players can be changed without changing the payoff to the strategies, then a game is symmetric. Many of the commonly studied 2×2 games are symmetric. The standard representations of chicken, the prisoner's dilemma, and the stag hunt are all symmetric games. Some scholars would consider certain asymmetric games as examples of these games as well. However, the most common payoffs for each of these games are symmetric.

Most commonly studied asymmetric games are games where there are not identical strategy sets for both players. For instance, the ultimatum game and similarly the dictator game have different strategies for each player. It is possible, however, for a game to have identical strategies for both players, yet be asymmetric. For example, the game pictured to the right is asymmetric despite having identical strategy sets for both players.

Zero sum games

They are a special case of constant sum games, in which choices by players can neither increase nor decrease the available resources. In zero-sum games the total benefit to all players in the game, for every combination of strategies, always adds to zero (more informally, a player benefits only at the expense of others). Poker exemplifies a zero-sum game (ignoring the possibility of the house's cut), because one wins exactly the amount one's opponents lose. Other zero sum games include matching pennies and most classical board games including Go and chess.

Many games studied by game theorists (including the famous prisoner's dilemma) are non-zero-sum games, because some outcomes have net results greater or less than zero. Informally, in non-zero-sum games, a gain by one player does not necessarily correspond with a loss by another.

Constant sum games

These correspond to activities like theft and gambling, but not to the fundamental economic situation in which there are potential gains from trade. It is possible to transform any game into a (possibly asymmetric) zero-sum game by adding an additional dummy player (often called "the board"), whose losses compensate the players' net winnings.

Simultaneous games

These are games where both players move simultaneously, or if they do not move simultaneously, the later players are unaware of the earlier players' actions (making them effectively simultaneous). Sequential games (or dynArtic games) are games where later players have some knowledge about earlier actions. This need not be perfect information about every action of earlier players; it might be very little knowledge. For instance, a player may know that an earlier player did not perform one particular action, while he does not know which of the other available actions the first player actually performed.

The difference between simultaneous and sequential games is captured in the different representations discussed above. Normal form is used to represent simultaneous games, and extensive form is used to represent sequential ones.

Perfect information and imperfect information

An important subset of sequential games consists of games of perfect information. A game is one of perfect information if all players know the moves previously made by all other players. Thus, only sequential games can be games of perfect information, since in simultaneous games not every player knows the actions of the others. Most games studied in game theory are imperfect information games, although there are some interesting examples of perfect information games, including the ultimatum game and centipede game. Perfect information games include mind games and chess. Perfect information is often confused with complete information, which is a similar concept. Complete information requires that every player knows the strategies and payoffs of the other players but not necessarily the actions.

Infinitely long games and Determinacy

Games, as studied by economists and real-world game players, are generally finished in a finite number of moves. Pure mathematicians are not so constrained, and set theorists in particular study games that last for infinitely many moves, with the winner (or other payoff) not known until after all those moves are completed.

The focus of attention is usually not so much on what is the best way to play such a game, but simply on whether one or the other player has a winning strategy. (It can be proven, using the axiom of choice, that there are games—even with perfect information, and where the only outcomes are "win" or "lose"—for which neither player has a winning strategy.) The existence of such strategies, for cleverly designed games, has important consequences in descriptive set theory. Application and challenges of game theory in one form or another are widely used in many different disciplines.

Roots of Game Theory in Political Economy

Political economy was the original term used for studying production and trade, and their relations with law, custom, and government, as well as with the distribution of national income and wealth. Political economy originated in moral philosophy a la Adam Smith. It was developed in the 18th century as the study of the economies of states, or polities, hence the term political economy. In the late 19th century, the term economics came to replace political economy, coinciding with the publication of an influential textbook by Alfred Marshall in 1890. Earlier, William Stanley Jevons, a proponent of mathematical methods applied to the subject, advocated economics for brevity and with the hope of the term becoming the recognized name of a science. William Jevons was one of three men to simultaneously advance the so-called marginal revolution. Working in complete independence of one another—Jevons in Manchester, England Leon Walras in Lausanne, Switzerland; and Carl Menger n Vienna—each scholar developed the theory of marginal utility to understand and explain consumer behavior. The theory held that the utility (value) of each additional unit of a commodity—the marginal utility—is less and less to the consumer. When you are thirsty, for example, you get great utility from a glass of water. Once your thirst is quenched, the second and third glasses are less and less appealing. Feeling waterlogged, you will eventually refuse water altogether. “Value,” said Jevons, “depends entirely upon utility.”

Today, political economy, where it is not used as a synonym for economics, may refer to very different things, including Marxian analysis, applied choice approaches emanating from the Chicago school and the Virginia school, or simply the advice given by economists to the government or public on general economic or on specific proposals. A rapidly growing mainstream literature from the 1970s has expanded beyond the model of economic policy in which planners maximize utility of a representative individual toward examining how political forces affect the choice of economic policies, especially as to distributional conflicts and political institutions. The question of rational choice is inscrutably woven into the fabric of the economic science ever since Bentham distinguished between subjective and objective conditions of rational behaviour. Political economy deals at a significant level with making decision choices under conditions of relative uncertainty. And as soon as this happens a condition is created for Game Theory to make its inroad.

Descriptive Version

A three stage Centipede Game. The centipede game, first introduced by Robert Rosenthal in 1981, is an extensive form game in which two players take turns choosing either to take a slightly larger share of a slowly increasing pot, or to pass the pot to the other player. The payoffs are arranged so that if one passes the pot to one's opponent and the opponent takes the pot on the next round, one receives slightly less than if one had taken the pot on this round. Although the traditional centipede game had a limit of 100 rounds (hence the name), any game with this structure but a different number of rounds is called a centipede game. The first use is to inform us about how actual human populations behave. Some scholars believe that by finding the equilibria of games they can predict how actual human populations will behave when confronted with situations analogous to the game being studied. This particular view of game theory has come under recent criticism. First, it is criticized because the assumptions made by game theorists are often violated. Game theorists may assume players always act rationally to maximize their wins (the Homo Economicus model), but real humans often act either irrationally, or act rationally to maximize the wins of some larger group of people (altruism). Game theorists respond by comparing their assumptions to those used in physics. Thus while their assumptions do not always hold, they can treat game theory as a reasonable scientific ideal akin to the models used by physicists. However, additional criticism of this use of game theory has been levied because some experiments have demonstrated that individuals do not play equilibrium strategies. For instance, in the centipede game, guess 2/3 of the average game, and the dictator game, people regularly do not play Nash equilibria. There is an ongoing debate regarding the importance of these experiments.

Alternatively, some authors claim that Nash equilibria do not provide predictions for human populations, but rather provide an explanation for why populations that play Nash equilibria remain in that state. However, the question of how populations reach those points remains open.

Some game theorists have turned to evolutionary game theory in order to resolve these worries. These models presume either no rationality or bounded rationality on the part of players. Despite the name, evolutionary game theory does not necessarily presume natural selection in the biological sense. Evolutionary game theory includes both biological as well as cultural evolution and also models of individual learning, (for example, fictitious play dynArtics).

The Prisoner's Dilemma

On the other hand, some scholars see game theory not as a predictive tool for the behaviour of human beings, but as a suggestion for how people ought to behave. Since Nash equilibrium of a game constitutes one's best response to the actions of the other players, playing a strategy that is part of Nash equilibrium seems appropriate. However, this use for game theory has also come under criticism. First, in some cases it is appropriate to play a non-equilibrium strategy if one expects others to play non-equilibrium strategies as well. For an example, see Guess 2/3 of the average.

Second, the Prisoner's Dilemma presents another potential counterexample. In the Prisoner's Dilemma, each player pursuing his own self-interest leads both players to be worse off than had they not pursued their own self-interests.

Biology and the hawk-dove game: Unlike economics, the payoffs for games in biology are often interpreted as corresponding to fitness. In addition, the focus has been less on equilibria that correspond to a notion of rationality, but rather on ones that would be maintained by evolutionary forces. The most well-known equilibrium in biology is known as the Evolutionary Stable Strategy or (ESS), and was first introduced by John Maynard Smith (described in his 1982 book). Although its initial motivation did not involve any of the mental requirements of the Nash equilibrium, every ESS is a Nash equilibrium.

In biology, game theory has been used to understand many different phenomena. It was first used to explain the evolution (and stability) of the approximate 1:1 sex ratios. Ronald Fisher (1930) suggested that the 1:1 sex ratios are a result of evolutionary forces acting on individuals who could be seen as trying to maximize their number of grandchildren.

Additionally, biologists have used evolutionary game theory and the ESS to explain the emergence of animal communication (Maynard Smith & Harper, 2003). The analysis of signalling games and other communication games has provided some insight into the evolution of communication among animals. For example, the mobbing behaviour of many species, in which a large number of prey animals attack a larger predator, seems to be an example of spontaneous emergent organization. Finally, biologists have used the hawk-dove game (also known as chicken) to analyze fighting behaviour and territoriality. In this 1982 book, the theory of games, first developed to analyse economic behaviour, is modified so that it can be applied to evolving populations. John Maynard Smith's concept of an evolutionarily stable strategy is relevant whenever the best thing for an animal or plant to do depends on what others are doing. The theory leads to testable predictions about the evolution of behaviour, of sex and genetic systems, and of growth and life history patterns. This book contains a full account of the theory, and of the data relevant to it. The account is aimed at senior undergraduate and graduate students, teachers and research workers in animal behaviour, population genetics and evolutionary biology. The book will also be of interest to mathematicians and game theorists; the mathematics has been largely confined to appendixes so that the main text may be easily followed by biologists.

Computer science and logic

Game theory has come to play an increasingly important role in logic and in computer science. Several logical theories have a basis in game semantics. In addition, computer scientists have used games to model interactive computations.

Separately, game theory has played a visible role in online algorithms. In particular, the k-server problem, which has in the past been referred to as games with moving costs and request-answer games. The k-server problem is a problem of theoretical computer science in the category of online algorithms, one of two abstract problems on metric spaces that are central to the theory of competitive analysis (the other being metrical task systems). In this problem, an online algorithm must control the movement of a set of k servers, represented as points in a metric space, and handle requests that are also in the form of points in the space. As each request arrives, the algorithm must determine which server to move to the requested point. The goal of the algorithm is to keep the total distance all servers move small, relative to the total distance the servers could have moved by an optimal adversary who knows in advance the entire sequence of requests.

ISSN: 2348 9510
International Journal Of Core Engineering & Management (IJCEM)
Volume 1, Issue 11, February 2015

Philosophy and the stag-hare hunt

Game theory has been put to several uses in philosophy. Responding to two papers by W.V.O. Quine (1960, 1967), David Lewis (1969) used game theory to develop a philosophical account of convention. In so doing, he provided the first analysis of common knowledge and employed it in analyzing play in coordination games. In addition, he first suggested that one can understand meaning in terms of signalling games. This later suggestion has been pursued by several philosophers since Lewis (Skyrms 1996, Grim et al. 2004).

In ethics, some authors have attempted to pursue the project, begun by Thomas Hobbes, of deriving morality from self-interest. Since games like the Prisoner's Dilemma present an apparent conflict between morality and self-interest, explaining why cooperation is required by self-interest is an important component of this project. This general strategy is a component of the general social contract view in political philosophy (for examples, see Gauthier 1987 and Kavka 1986).

Other authors have attempted to use evolutionary game theory in order to explain the emergence of human attitudes about morality and corresponding animal behaviours. These authors look at several games including the Prisoner's Dilemma, Stag hunt, and the Nash bargaining game as providing an explanation for the emergence of attitudes about morality (see, e.g., Skyrms 1996, 2004; Sober and Wilson 1999).

For the student of economic science Utility is usefulness, the ability of something to satisfy needs or wants. Utility is an important concept in economics and game theory, because it represents satisfaction experienced by the consumer of a good. Not coincidentally, a good is something that satisfies human wants and provides utility, for example, to a consumer making a purchase. It was recognized that one can not directly measure benefit, satisfaction or happiness from a good or service, so instead economists have devised ways of representing and measuring utility in terms of economic choices that can be counted. Economists have attempted to perfect highly abstract methods of comparing utilities by observing and calculating economic choices. In the simplest sense, economists consider utility to be revealed in people's willingness to pay different amounts for different goods. And it is here that Game Theory makes an exalted entry.

Conclusion

What economists call game theory psychologists call the theory of social situations, which is an accurate description of what game theory is about. Although game theory is relevant to parlour games such as poker or bridge, most research in game theory focuses on how groups of people interact. There are two main branches of game theory: cooperative and non-cooperative game theory. Non-cooperative game theory deals largely with how intelligent individuals interact with

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Volume 1, Issue 11, February 2015

one another in an effort to achieve their own goals. And it is here that students of Management Strategy and Social Sciences shake hands with the students of Quantitative Methods at the post-graduate level. This paper has attempted to posit various ramifications of this basic idea in simple to understand and non-mathematical language. However in view of the mutual nature of social reality and accompanying polity as well as the dynamics of power the scope for using game theory is infinite and this is where some of the future research should, in my opinion, head.

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