

A REVIEW ON VORTEX FLOW OVER DELTA WINGS AT HIGH ANGLES OF ATTACK

Salaheldin H. Omar

*Professor of Aerospace and Autonomous Systems Engineering and former Chair;
Talent and Technology Creativity Unit,
University of Tabuk, Saudi Arabia.
salaheldin.omar@outlook.com*

Abstract

Vortex flow is an important phenomenon associated with high agility aircrafts at super manoeuvring rates, slender body of revolutions, highly swept back and delta wings, dynamical meteorology, physical oceanography and enhanced fuel/air mixing process inside the internal combustion chambers. This review covers the vortex flow physics and phenomena since 1858 up to the recent developments with special attention on these phenomena on delta wings covering the physics of vortex initiation, development and breakdown. The review deals with the development of the research and physics of vortex initiation, development and breakdown. Different perspectives and explanations of vortex phenomena and various types of vortex breakdown have been considered.

Keywords: Vortex Flow, Vortex Breakdown, Vortex Dynamics, Delta Wing, Super Manoeuvring Aircraft, High Agility Aircraft, Slender Body of Revolution, Wind Tunnel, Pressure Measurements, Delta Wing Structure Deformation, Data Acquisition Systems.

Nomenclatures

Cr - Wing root chord

Re - Reynolds number based on chord length of the wing

Λ - Aspect ratio

r - Radius of viscous sub core

λ - wave length

ρ - Fluid density

ρ_0 . Reference density (a pure constant)

k - Number of waves

δ - Local vortices thickness (vertical extent of mixing zone)

$\Delta \rho$ - Density difference of two adjacent fluid streams

ΔU - Difference between flow Velocities of two adjacent fluid streams

ω - Angular velocity

R - Radial distance from the centre of the vortex core

$g' = g (\rho_2 - \rho_1)$ is the reduced gravity

S - Local wing span

U_∞ - Free stream speed

x - Local chord-wise distance from wing apex

y - Local span-wise distance from wing root “from wing line of symmetry”
Z - Local distance above wing surface
h - Maximum wing profile thickness
 α - Wing angle of attack
 α_C - Vortex-core angle of attack
 Λ_C - Vortex-core sweep angle
 β - Wing angle side slip
 Λ_{LE} - Wing sweep back angle
Vswirl - Vortex swirl velocity
Vaxial - Vortex axial velocity
 ϕ - Swirl angle = $\tan^{-1} (V_{swirl} / V_{axial})$
Cp - Surface pressure coefficient

I. INTRODUCTION

Vortex flow is an important phenomenon associated with numerous of fields and applications as, super manoeuvring or high agility aircrafts, slender body of revolutions, highly swept back wings and delta wing, dynamical meteorology. It covers thunderstorms of few km length scale and velocity of 1-10 m/s and lasts for few hours, sea breeze of 5-50 km length scale, velocity of 1-10 m/s and last for 6 hours, tornado of 10-500 m length scale, velocity of 30-100 m/s velocity and last for 10-60 minutes, hurricane of 300-500 km length scale, velocity of 30-60 m/s and last for days to weeks, mountain waves, and physical oceanography including oceanic waves, vortices, currents, long oceanic waves, atmospheric convection, and natural oscillations in air/sea interactions (Philander, S. G. (1990) [1]; D'Aleo, J. S. (2002) [2]). It enhances the mixing process of injected fuel inside the engine combustion chambers and increasing the combustion performance and efficiency reducing the fuel consumption (Banea, S.P.M et al (2015) [3]). Leeward vortices over solid objects tends to become asymmetric and to breakdown in an unsteady spiral or axis symmetric form especially for slender delta wings and slender bodies of revolutions at high angles of attack. Asymmetrical and ant symmetrical vortex flow and breakdown affect fighter airplanes manoeuvring at high angles of attack and sideslip. Anti symmetrical vortex flow results in large side forces and moments beyond the maximum possible moments of the control surfaces at their maximum deflection positions which reduces the lateral stability and controllability of the aircraft. At high angles of attack, the vortex breakdown location moves upstream close to the apex affecting the majority of the wing area and the aircraft aerodynamic buffeting is very likely to occur (Gursul, I. (1994) [4]).

II. THE VORTEX FLOW

2.1. The Vortex Flow and the primary vortex;

The presence of two separated parallel streams of different velocities or at the leading edge of delta wings results in developing of a mixing layer. The first stream of higher velocity comes from the leeward side of the wing while the other from the pressure side. Mixing layers are growing initiating big spiral vortices due to the known Kelvin-Helmholtz type instability. Helmholtz have grounded the

science of vortex dynamics in 1858. He investigated, among others, the vortex flow and he was the first who investigated the vortices interacting with solid boundaries, he investigated axi symmetrical flow of multi circular vortex-filaments and the mutual interactions of dual vortices. He developed also his known four known vortices theories [5] Helmholtz, H. (1858). The First Theorem of Helmholtz, H. (1858) [5] states that “the infinitesimal fluid particle centred at position (x, y, z) with velocity components (u, v, w) moves like a solid body having translational velocity (u, v, w) and a rotational velocity induced by shearing forces, which generates deformation velocities superposed on the original velocities leading to a sudden stop followed by an angular velocity of one-half the fluid vorticity”. The Second Theorem of Helmholtz, H. (1858) [5] states that “for a vortex filament, the product of the magnitude of the vortices and the cross-sectional area remains a constant called intensity of the filament. The vortex filament keeps a conserved vorticity and consequently it cannot vanish in the interior of the fluid but instead it forces the formation of closed rings or it vanish there on the boundary”. The Third Theorem of Helmholtz, H. (1858) [5] states that, “the fluid which form a vortex tube continues to form a vortex tube”. The Fourth Theorem of Helmholtz, H. (1858) [5] states that, “the intensity of a vortex tube remains constant as the tube moves about. Accelerated Flow in the region between the crests and troughs results in mass conservation in the perturbations similar to waves”.

Interfacial waves results in mixing of fluids with growing perturbations during the fluid flow at intermediate speed leading to vertical motions and overturning which form the final primary vortex stage over delta wings. The waves/flow local coupling mechanism extracts enough energy from the flow to propagate the perturbations. In a homogeneous fluid, mixing requires enough energy necessary to overcome mechanical frictions, but in a stratified fluid, additional energy is required to raise heavy fluid parcels up and lower light fluid parcels down resulting in an increase in the potential energy. If the decrease in the kinetic energy is greater than the increase in the potential energy, mixing process would continue in a spontaneous manner.

Mixing of fluid parcels can be carried out only under two conditions, first if the difference in the initial density is too small to avoid the barrier of the gravity (Thorpe, S. A. (1968, 1971) [6, 7]) and second if the initial velocity-shear is large enough to generate the required energy to make the fluid mixing process possible. Richardson number “Ri” determines the ratio between potential and kinetic energies, the numerator is the potential-energy barrier to be overcome if the mixing process is to be performed and the denominator is the kinetic energy in the shear flow.

$$Ri = \frac{(g/\rho_0)(\rho_2 - \rho_1)\delta}{(U_1 - U_2)^2} \quad \text{Richardson Lewis Fry (1922) [8]}$$

U_1 , U_2 and ρ_2 , ρ_1 are the upper and lower densities and velocities respectively, δ is the local vorticity thickness (vertical extent of mixing zone) and ρ_0 is the reference density (a pure constant). Kelvin-Helmholtz instability was first studied by Taylor in 1915 [9] Taylor, G. I., (1931) concluding that Richardson number must be less than $\frac{1}{4}$ to initiate instability, otherwise, the mixing occurs only locally in a vicinity of the initial interface and will not have the capability to propagate over the whole system. The perturbations of all wavelengths must be considered to investigate the initiation of stability leading to the fact that the perturbations of short wavelength are always present and therefore, a two-layer shear flow is always unstable “Kelvin-Helmholtz instability (KHI)”.

$$((\rho_2)^2 - (\rho_1)^2)g' < \rho_1\rho_2 k (U_1 - U_2)^2 \text{ Kundu, V.; Cohen I. M. (1971). [10]}$$

Where k is the wavelength and $g' = g(\rho_2 - \rho_1)$ is the reduced gravity

For flows of small density differences " $(\rho_1 \approx \rho_2 \approx \rho_0)$ ", the Boussinesq approximation is used as;

$2(\rho_2 - \rho_1) g < \rho_0 k (U_1 - U_2)^2$ and Richardson number will take the form;

$Ri = (g / \rho_0) (\rho_2 - \rho_1) \delta / (U_1 - U_2)^2$, which is also used for atmospheric or oceanic flows.

Close neighbored vortices tend to pair, rotate around each other's and merge under the effect of their induced velocities. Each vorticity tends to entrain the other vorticity by its induced velocity forcing both vortices to rotate around one another in pair with angular velocity $\omega \propto R^{-2}$ where R is the radial distance from the centre of the vortex core. The vorticity rotates slower at the outer part than close to the centre developing vorticity tails as shown in figure 1-C which merge into spirals during the pairing process. The wavelength of maximum amplitudes is dependent on the condition of the initial mixing layer profiles, it ranges from 6 to 11 times the initial thickness of the transition layer. The interfacial waves induce flow perturbations extending in both sides of the interface to heights in the order of their wavelength $\lambda = 2 \rho / k$, overturn at similar vertical and lateral wave scales. Mixing layers are growing initiating big spiral vortices due to the known Kelvin-Helmholtz type instability. Experiments of Winant, C. D.; Browand, F. K. (1974) [11] and Brown, G. L.; Roshko, A. (1974) [12] predicted rolled up vortices at both low and high Reynolds numbers. Winter H. (1935, 1936) [13, 14] was the first who investigated the vortex flow at the wing leading edge using different flow visualization techniques (smoke, tufts, and "soot and petroleum) followed investigations on a delta wing using tuft analysis by Wilson, H. A. Jr.; Lovell, J. C (1947) [15] who predicted vortices over the leeward surface of low aspect ratio delta wings including those of sharp edges.

Örnberg T. A. (1954) [16] identified the secondary vortices initiated by the separation of the leeward surface boundary layer. Werle H. (1954) [17] visualized the vortex and explained the vortex breakdown as an expansion of the free, spiral-shaped vortex due to the transition from laminar to turbulent flow. Peckham D. H. and Atkinson S. A. (1957) [18] and Peckham D. (1958) [19] have similar results. Elle B. J. (1958) [20] studied the location of a vortex over sharp edged delta wings in both water and air, defined the vortex breakdown as a failure of the vorticity structure to maintain its form around the vortex core. They have discovered the sudden expansion of the cross-sectional area of a leading-edge vortex core by increasing the incidence beyond a critical angle, which depends on the wing sweep angle. Bergesen, A. J.; Porter J. D. (1960) [21] described the initial movement of the stagnation point from the leading edge toward the pressure side of the delta wing, as the angle of attack increases from zero, and proceeds from there towards the leading edge under the induction of the suction pressure on the leeward side of the wing. The centrifugal forces acting on the fluid elements drawn by the suction pressure on the leeward side of the sharp edged delta wing around its leading edge approaches infinity and cannot remain attached to the surface and separates at the sharp leading edge forming vortex sheet growing in strength by proceeding inboard and downstream rolling up to a conical vortex. Weber, J. (1955) [22] and Marsden, D. J. et al (1957) [23] explains the extra lift on delta wings as an effect of extra entrainment of the air forced by the leading edge vortices. They have attributed the nonlinearity in the extra lift to the inward and upward movements of the vortex core and to the increase of the vortex strength due to the increased angle of attack leading to the propagation of the effect of entrainment.

Hummel, D. (1965) [24] and Hummel D. & Srinivasan P. S. (1967) [25] have investigated the vortex flow and breakdown over delta wings including pressure measurements and flow visualization. Michalke A. (1965) [26] used the nonlinear theory of Stuart J. T. (1961)[27] to investigate the initiation and development of vortices, while Winant C. D. et al (1974) [11] did similar investigation experimentally. Jones, J.P., (1960) [28] and Ludwig, H. (1961, 1962, 1965) [29, 30, 31] estimated in their hydrodynamic instability that sensitivity of the vortex core to spirals dominates the axial effects in developing a stagnation point. Squire, H. B., (1960) [32] stated that standing waves with axial periodicity can exist in cylindrical vortex motion if the flow is subcritical. In a sub critical flow disturbances can propagate up and downstream and standing waves are supported, whereas in a supercritical flow only downstream propagation is possible. Standing waves might be formed in a sub critical flow in case of upstream propagating disturbances leading to vortex breakdown.

According to Benjamin, T. B. (1962, 1965, 1967) [33, 34, 35], vortex breakdown is a transition state having finite amplitude between two dynamically conjugate flow states. A cylindrical, in viscid, and supercritical flow developed to a subcritical state backing up the axisymmetric standing waves with finite amplitude. Experimental investigations of Sarpkaya T. (1971) [35] predicted significant influence of the boundary conditions on the development and on the vortex structure of vortex breakdown, which was not considered in KHI and other theories. Delta wings have two flow streams of both lee and pressure sides, which join at the separation line along the wing leading edge forming a separated free shear layer of two separated adjacent streams of different velocities, namely the lower velocity stream from the lower pressure side, and the higher velocity stream from the suction leeside. The different velocities of the two parallel streams of the separated shear layer initiate in viscid instable layer of a constant vorticity in the interfacial region inside the free shear layer and consequently laminar small waves are created leading to the distortion of the boundaries of the region containing the vortices as in figure 1(a).

The perturbations induce vertical velocities which force the perturbations and waves to propagate into discrete vortices and therefore to increase the thickness of the free shear layer as they moved away from the leading edge, figure 1 (b). Mixing layers are growing initiating big spiral vortices due to the known Kelvin-Helmholtz type instability as mentioned before. Each vorticity tends to entrain the other neighbored vorticity under the influence of their induced velocity forcing both vortices to rotate around one another in pair due to the nonlinear interaction. Every vorticity rotate slower at its outer part than close to the centre initiating tails as shown in figure 1-C which merge into spirals during the pairing Piercy N. A. V. (1923) [36], Winant, C. et al (1974) [11].

The interfacial region of constant vorticity becomes periodically fatter and thinner) and the pairing process starts. The pairing process amplifies the spatial and irregularities in the vortex structure and increase the variations in the length and strength of the vortex cores which are continually rotate around one another in pairs forming the rotating vortex lumps in the interfacial region of the free shear layer as in figures 1(d) and 2. The free shear layer rolls up into a vortex core by the impact of the vorticity induced flow velocities on both sides of vortex lumps within the free shear layer as shown in figure 1- (d and e) and figure 2. It is also obvious in figure 3 of wind tunnel smoke and laser light sheet flow visualization over sharp edged delta wing after Omar, Salaheldin H. et al (2020) [37]. The primary vortex can be divided, after Earnshaw P. (1962) [38], into three

regions, namely; the free shear layer, the rotational core, and viscous sub core as shown in figure 2.

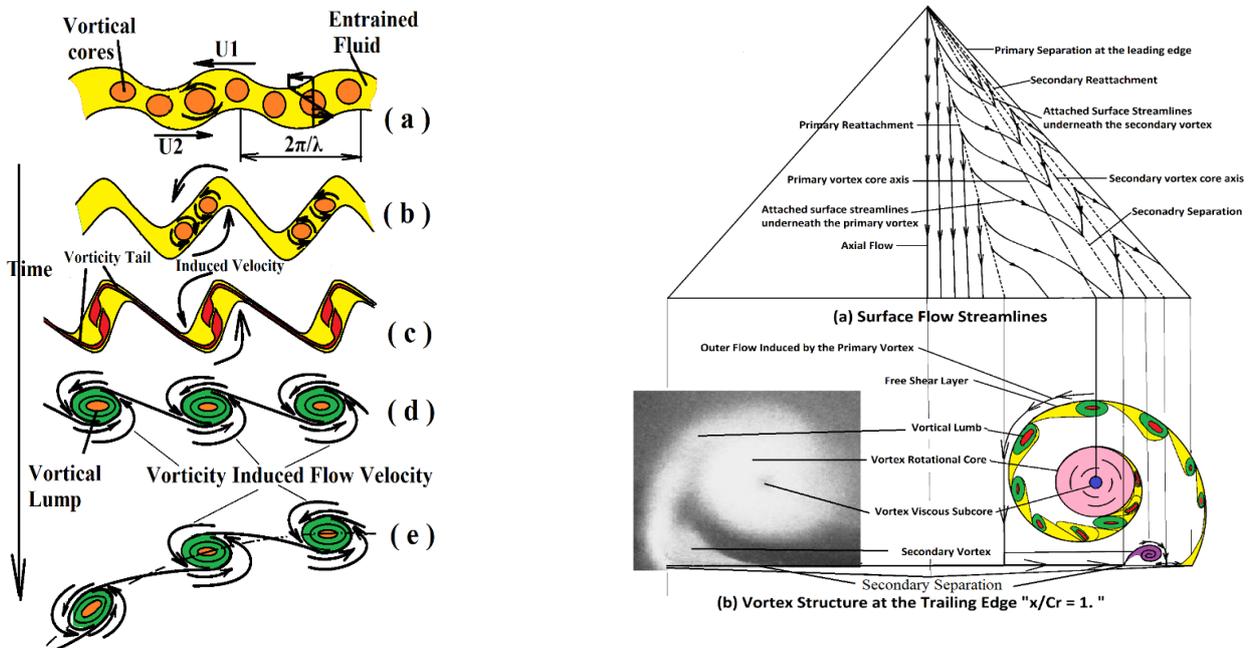


Figure 1 (a-e): Vortex pairing and lumps Figure 2: surface flow stream lines and vortex flow Of a delta wing

The outer flow induced by the primary vortex rotates around it and reattaches on the leeside surface of the wing. The region between the two reattachment lines of the primary vortices is dominated by axial flow, while the region between these two reattachment lines and the leading edge is dominated by cross flow as in figure 2 (a).

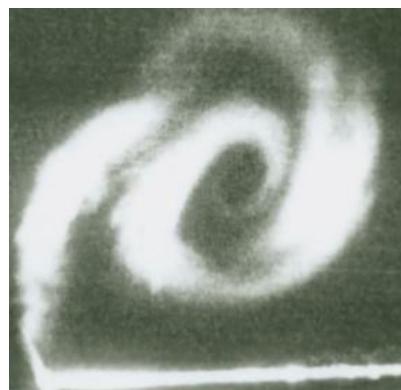


Figure 3: Laser smoke and light sheet illumination of the cross flow on a sharp edged delta wing, Omar, Salaheldin H. et al (2020)[37]

2.2. The secondary Vortex

The secondary vortices produced by the separation of the upper surface boundary layer, shown in figure 2, have been identified by Örnberg T. (1954) [16]. The free shear layer generated at the leading edge rolled up forming the primary vortex induces the outer flow of the primary vortex to reattach on the leeward side of the delta wing forming reattachment line extending from the apex region to the trailing edge and is continually providing the boundary layer after the reattachment line with fresh air of high energy as illustrated in figure 2. The reattached flow moves from the reattachment line outboard towards the leading edge until it separates at the secondary separation line somewhere between the axis of the primary vortex and the leading edge in dependence of the flow condition “laminar/turbulent” forming the secondary vortex. Tertiary vortex may be initiated underneath the secondary vortex with a rotation in the same sense of the primary vortex.

2.3. Laminar / turbulent transition

At higher angle of attack and Reynolds number, a laminar/turbulent transition of the boundary layer in the rear part of the wing provides the reattached flow with higher energy maintaining the flow attached longer distance toward the leading edge at and aft this location forming outboard kink of the secondary separation line at this location and decreasing its distance from the leading edge as in figure 4 and consequently, the possibility of initiation of tertiary and multiple vortices becomes very hard. A secondary vortex, smaller in size and lower in strength than in the case of laminar boundary layer, is formed leading to greater upward and inward displacements of the primary vortex than for laminar boundary layer Payne F. M. et al, (1986) [39], Payne F. M., (1987) [40]. The relation between lift and angle of attack is linear for wings of high aspect ratios and nonlinear for wings of low aspect ratios. The nonlinearity, which is initiated by the vortex sheet above the wing, increases with decreasing the aspect ratio.

The circulation increases with angle of attack increasing the lift-curve slope and decreases by increasing the sweep angle reducing the lift-curve slope. The secondary vortex undergoes changes, resembling the breakdown phenomena of the primary vortex before it occurs in the primary vortex. The secondary vortex retains its vortex flow structure aft of primary vortex breakdown location Nelson R. C. and Visser K. (1991) [41]. Increasing the Reynolds number results in increasing the unsteady perturbations in the shear layer Riley A. J. and Lawson M. V. (1998) [42]. Delery J. (1994) [43] suggested that the secondary vortex is not affected by the breakdown of the primary vortex. Huang, X. Z., and Hanff, E. S. (1998) [44] relates the outboard kink of the secondary separation line to the vortex breakdown, Reverse flow patterns were observed when breakdown was in the apex region. Earnshaw P. B. and Lawford J. A. (1964) [45] visualized a “whorl” close to each leading edge in the rear part of the wing.

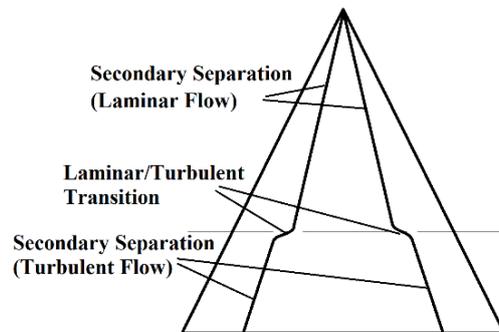


Figure 4: Laminar/turbulent transition of the flow over delta wing

2.4. Vortex Breakdown

2.4.1. Vortex Breakdown Phenomena

The vortex breakdown has a very complicated structures and mechanisms with many still unresolved problems and contradictory explained phenomena. Vortex breakdown can be beneficially used in many applications as for enhancing the mixing process of injected fuel inside the engine combustion chambers (Banea, S.P.M.; Zieglerb , J.L. (2015) [3]) or it could have negative and severe effects and consequences on the aerodynamic forces and performance of aircrafts and might lead to catastrophic interactions among aerodynamic forces, inertia forces and forces from the structure elastic deformation (Bisplinghoff, R. et al (1955) [46], Ashly H. et al (1967) [47]) as it is the case for the F18 aircraft.

Vortex breakdown has many different and contradictory explanations, aspects of view, and interpretations as; a consequence of hydrodynamic instability (Ludwig, H. (1961, 1962, 1965) [29, 30, 31]), a similar form to the separation of a two-dimensional boundary layer (Hall, M. G. (1966) [48], Mager, A. (1972) [49]), a solitary wave resulted from long propagations of trapped weak nonlinear waves in nearly critical swirling flows (Leibovich, S. (1978) [50], Randall, J. D. and Leibovich, S. [51]), or as a development of a critical state representing a finite transition between states, similar to a hydraulic jump Squire, H. B., (1960) [32], Benjamin, T. B. (1962, 1965, 1967) [33, 34, 35].

Vortex breakdown was first discovered by Werle H. (1954) [17], who visualized the vortex and explained the vortex breakdown as an expansion of the free, spiral-formed vortex due to the transition from laminar to turbulent flow. Vortex breakdown was confirmed by further investigations from Peckham, D. H. et al (1957) [18] and Peckham, D. H. (1958) [19], Elle, B., J. (1958) [20], Bergesen, A. J.; Porter J. D. (1960) [21]. The first theoretical explanation of the vortex breakdown is the theory of Jones, J.P. (1960) [28] and Ludwig, H. (1961) [29], who relate the vortex breakdown to the hydrodynamic instability of the vortex core with respect to asymmetric spiral disturbances supported by the prediction of the helical breakdown, while this theory is not valid for the axisymmetric bubble form breakdown.

Vortex breakdown was related by Squire, H. B. (1960) [32], Benjamin, T.B., (1962) [33] to the capability of the downstream disturbances to propagate upstream and disrupt the flow. Harvey J. (1962) [52] observed the smoke-visualized solitary vortices in transparent tubes and predicted only

bubble form vortex breakdown with partially disorganized flow downstream to it, which could be restored to a normal vortex in appropriate condition, he came to the conclusion that the vortex breakdown is a transition of two kind of organized swirling flows. The theoretical explanations of the vortex breakdown by Benjamin, T.B., (1962) [33] is based on the experimental results of Harvey J. (1962) [52]. Benjamin, T.B., (1962) [33] opposed the explanation of the vortex breakdown as the result of the hydrodynamic instability of Jones, J.P., (1960) [28], Ludwig, H. (1961) [29] because the generated vortices by Harvey J. (1962) [52] were stable and axially symmetric upstream of the breakdown location. Benjamin, T.B., (1962) [33] explained the vortex breakdown as a hydraulic jump at which the flows upstream and downstream to it were in conjugate states and based on the flow condition, small axisymmetric dispersive waves could not propagate upstream (supercritical) and in other condition could propagate upstream and downstream (subcritical), he defined the vortex swirl by the ratio of azimuthally to axial velocity. The theoretical explanations of the vortex breakdown by Hall M. G. (1961) [53] has considered the vortex upstream to the breakdown as a quasi-cylindrical form of much smaller axial gradients than the radial gradients in the vortex core and investigated numerically the downstream development of the vortex for many flow conditions. Some calculations of some conditions fail to converge.

Lambourne and Bryer (1961) [54] discovered both spiral and vortex breakdown forms and observed the suddenly unexpectedly change in the structure of the uniform vortex core over a delta wing into helical and bubble forms vortex breakdown followed by disordered, wavelike flow aft the vortex breakdown location. Lambourne and Bryer (1961) [54] stated that vortex bursting involves a sudden decrease in the magnitude of the axial and circumferential velocity components of the core. Similar investigation were accomplished by Hall M. G. (1961) [53], Ludwig, H. (1962) [30]. Vortex breakdown is defined by Sarpkaya T. (1971) [35] as sudden sharp deformation of the vortex core structure resulting in expanding asymmetric flow around the vortex axis. Leibovich S. (1978) [50] and Garg A. & Leibovich S. (1979) [55] described vortex breakdown as a disturbance which decelerates the internal flow along the vortex axis up to a stagnation condition followed by reversing the flow direction.

Vortex breakdown occurs, after Schade H., Michalke, A. (1962) [56] in dependence of the external pressure gradient and the degree of the divergence as indication of the vorticity convection along the vortex axis, and after Hall M. (1972) [57], it occurs in dependence of the magnitude of the flow swirling, as an indication of the vorticity shedded in the rolled up free shear layer. The higher adverse pressure gradient and the low degree of divergence along the axis of the viscous sub core at high angles of attack reduce the swirling flow of the vortex core, therefore, the vorticity balance concept is an important explanation of the vortex breakdown over delta wings, it states that, a stable leading edge vortex can be maintained only if the vorticity generated at the leading edge and shedded inside the rolled up free shear layer "vortex sheet" is balanced by enough convection of the vortices along the vortex core axis for both axisymmetric or non-axisymmetric vortex breakdown conditions (Reynolds, W.C.; Carr, L. W. 1985 [58]).

A swirl angle $\phi = \tan^{-1} (V_{swirl} / V_{axial})$ degree is an indicator of the vorticity balance condition Lee M., and Ho C. (1989) [59]. High swirl angles generally greater than 40° are observed just upstream to the breakdown location. At a certain angle of attack the increased rate of generation of the vortices shedded in the vortex sheet exceeds the convection of these vortices leading to to an

increase in the concentrated vortices, which have limits on their maximum amount of vortices per unit area "critical vortices concentration". By exceeding these limits, the shedded vortices in the rolled up free shear layer cannot be compensated by the convection of the vortices, which is dependent on the increased component of the axial flow velocity by increasing the angle of attack, and gradually, the axial momentum becomes too weak to overcome the adverse pressure gradient, leading to a drastic increase in the interactions among the vortex-outer-core spirals, and to the formation of a stagnation point along the viscous sub core, subsequently, the vortex cannot maintain its organized structure leading to a spiral form or bubble form vortex breakdown spreading the vortices over a wider region reducing the vorticity concentration and the excessive vortices are redistributed in the region aft the vortex breakdown location. Vorticity concentration is dependent on many parameters such as the effective angle of attack, sweep angle, free-stream speed, airfoil geometry, and Reynolds number. Vortex burst progression is most rapid in the rear part of the wing, at which the adverse pressure gradient is maximum, and is slow close to the apex, at which the bursting location is far from the influence of the adverse pressure gradient aft the trailing edge independent of the Reynolds number Elle B. (1958) [60] and Lee M. and Ho C. M. (1990) [61].

The primary vortex is insensitive to Reynolds number for sharp edged wings because the separation is fixed along the leading edge, while the location of the secondary separation and the strength of the secondary vortex on the leeside surface are dependent on the Reynolds number Erickson G. (1982) [62], and Lee G. (1955) [63]. Payne F. et al (1987) [40] predicted experimentally a rapid expansion of the vortex core region and a disorganized flow after the vortex breakdown location. Nelson R. & Visser K. (1991) [41], Visser K. et al 1993 [64] and Visser K. et al (2004) [65] described the vortex breakdown as a drastic change in the flow field due to the sudden increase in the axial pressure gradient in the vortex core. Wang, J.; Zhan J. (2005) [66] predicted in water tunnel experiments at low Reynolds numbers sudden expansion and unsteadiness of the vortex core. Harvey J. (1962) [52] stated that bubble form vortex breakdown always occurs near the critical state. Lawson M. G. (1964) [67] was the first who mentioned that the final position of an established vortex breakdown is governed by different parameters from those governed its initial formation and that the final breakdown location depends on the adverse pressure gradient affected by the flow at the trailing edge.

The unsteady mixing in the breakdown zone dissipates vorticity and causes a turbulent wake as well as loss of lift. Greenwell D. & Wood N. (1994) [68] investigated the flow after vortex breakdown and divided the separated flow over the delta wing into 4 regions as follows; 1) Unburst vortex or primary core, 2) Flow deceleration region, 3) Bubble formation and 4) Fully developed breakdown area. The formation and behaviour of the primary core (region 1) has already been discussed above. Further aft of the apex (region 2), the flow decelerates under the influence of the adverse pressure gradient within the vortex core, this occurs in the second region, where the swirling magnitude of the primary vortex reduces. The flow field in this region cannot retain the vortex shape and the swirling magnitude is reduced further downstream. In region 3, a further reduction in the swirling magnitude is accompanied by the formation of a bubble like structure with some degree of reversed flow around the vortex core. In region 4, fully developed vortex breakdown with large scale turbulent is found and the flow becomes very unsteady and asymmetric.

Vortex breakdown moves upstream towards the apex by increasing the angle of attack until certain angle, at which this movement slows down as a result of the decreased adverse pressure gradient (close to zero) along the vortex core near to the apex and the vortices become stronger in this region and more able to resist the forward upstream movement of the vortex breakdown Wentz W. & Kohlman D., (1971) [69]. The leading-edge vortex bursts at a certain location far downstream of the trailing edge at low angles of attack. By increasing the angle of attack, the lift of slender sharp-edged delta wing increases nonlinearly up to a certain angle, at which the vortex bursting location reaches the trailing edge. These results in decreasing the lift, lift induced drag, and nose-down pitching moment.

Vortex bursting affects the pitching moment more than the lift, because it influences mainly the rear part of the wing. By increasing the aspect ratio vortex breakdown occurs at lower angle of attack and the changes in lift curve slopes and pitching moment decrease Wood, R. M.; Miller, D. S (1985) [70]. For very slender wings at high angles of attack the leading-edge outer vortex sheets of both sides meet at the centreline prior to the vortex bursting. At angle of attack above initial contact, the vortex cores are forcibly displaced upward leading to vortex-lift reduction. For delta wings with sweep angle of 70° or less, the bursting of the vortex at the trailing edge is not well correlated with the maximum lift condition and does not result in the onset of stall, or even significant loss of lift Wentz, W. H. Jr (1968) [71].

The primary effect of burst appears to be slight reduction of the local growth rate of the vortex. The leading edge shape significantly affects the bursting location, even for thin flat-plate delta wings. The characteristic of separation near the leading edge and the strength of the leading edge vortex seems to be the most important features for determining the vortex lift. The strength of the primary vortex increases with increasing the angle of attack and the axial velocity in the vortex core can exceed three times the free stream speed. The stability of the primary vortex depends on the balance process between the vorticity shedding and the convection along the axis of the viscous subcore Payne F. et al, (1986) [39], Payne F. (1987) [40] and the unbalance of this process leads to vortex breakdown.

Vortex breakdown hinders aircraft manoeuvrability and limits its performance at higher angles of attack, which affects substantially the characteristics of the aircraft stability due to the loss of lift and the substantial changes in pitching moment. The increase in turbulent intensity of the downstream flow also adversely affects the control surfaces such as the fin and rudder of fighter aircraft Payne F. et al (1986) [39]. Increased leading-edge sweep decreases the leading-edge vortex strength Hemsch, M. J. and Luckring, J. M (1990) [72] and delays vortex breakdown.

Wentz, W. H. and Kohlman, D. L (1971) [69] found that for sweep angles higher than 75° , breakdown becomes independent of the sweep angle. Furthermore, the coupling between vortex burst and lift is not strong for wings with sweep angles of 70° or less Kegelman, J. and Roos, F. (1989) [73], i.e. the first occurrence of vortex breakdown over a delta wing does not correspond to maximum lift, as commonly assumed in numerous textbooks. The primary effect of burst rather appears to be a slight reduction of the local growth rate of the vortex. The vortex breakdown is affected by the vortex core angle of attack and vortex core sweep angle. The vortex core sweep

increase by increasing the wing leading edge sweep Λ_{LE} and the vortex core angle of attack α increases by increasing the wing angle of attack Erickson, G. E. (1982) [62].

2.4.2 Types of Vortex Breakdown

Many different types of vortex breakdown have been identified in vortex tube experiments. Lambourne N. C., and Bryer D. W. (1961) [54] were among the first who investigated and defined the types of vortex breakdown on a delta wing as bubble and spiral vortex breakdown. For slender wings at angle of attack only two types of breakdown are generally identified, the bubble and the spiral breakdown, although in reality they may just represent the extremes of the breakdown forms. Vortex breakdown location is not a function of the Reynolds number for sharp edged.

2.4.2.1. Spiral breakdown

Lambourne N. C., and Bryer D. W. (1961) [54] characterized the spiral vortex breakdown by a rapid deceleration of the core flow followed by an abrupt kink at which point, the core flow takes the form of spirals making one or two turns before breaking up into large scale turbulence as shown in figures 5 and 6. The breakdown is due to the fact that high velocity vortex core cannot maintain its straight course. The vortex core is, therefore, forced to move outward in an effort to maneuver its way downstream. This outward motion takes the form of spirals in a direction opposite to the original motion of the upstream vortex, however the winding rotates in the same direction as that of the upstream vortex because the motion of the rotational outer core is consistent with the original sign of vorticity and because of the need to conserve the angular momentum of the fluid particles within the core. Another explanation by Ludwig H. (1962, 1965) [30,31] stated that spiral form of breakdown can be explained as a hydrodynamic instability of the approach flow against small helical disturbances, which means that there is at least two different mechanisms for vortex breakdown. Vortex breakdown cannot be explained by one theory only. Spiral vortex breakdown is a result of instability of the viscous sub core and not the outer vortex layers. Spiral breakdown occurs primarily if the breakdown position is approaching the trailing edge Lambourne N. & Bryer D. (1961) [54], Payne, F. M. (1987) [40] and Faler, J. H.; Leibovich, S. (1977) [76].

2.4.2.2. Bubble breakdown

Bubble breakdown is characterized by a stagnation point on the vortex axis followed by an oval shaped recirculation zone Payne, F. M. et al (1986) [39]. The upstream half of the recirculation zone is nearly axisymmetric with the flow passing around it, however the downstream half is usually open and irregular with the flow shedding from the aft end as if it were shedding from a blunt solid body in the form of shedded rings. The bubble length is usually two to three times that of the upstream-core diameter as in figure 5, 6. Downstream of the bubble the vortex is turbulent and diffuses rapidly with downstream movement. Bubble breakdown occurs only if the breakdown location is in moving condition close to the apex Payne, F. M. et al (1986) [39], Payne F. (1987) [40] and Faler, J. H.; Leibovich, S. (1977, 1978) [74, 75].

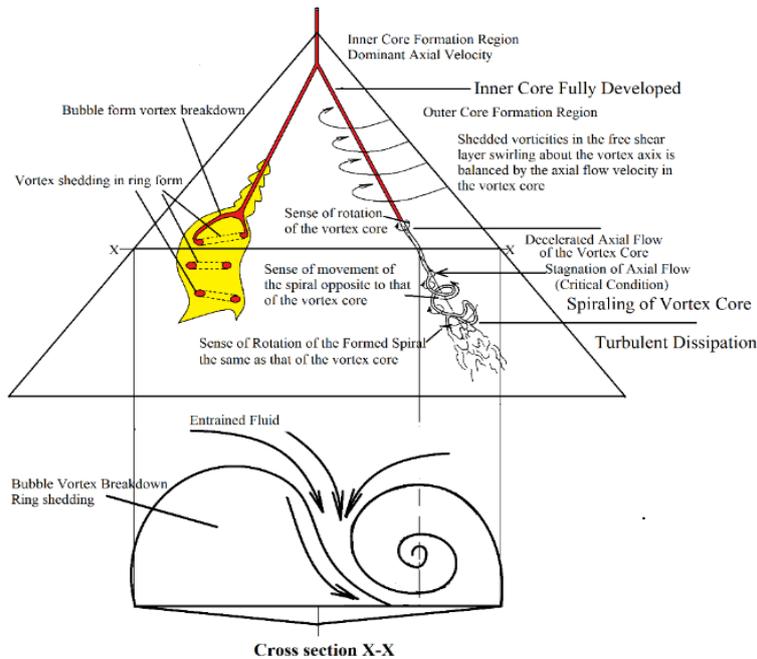


Figure 5: Vortex formation and breakdown over delta wings

2.5 Unsteady vortex flow

Unsteady vortex flow and unsteady vortex flow phenomena, which might induce wing or wing/tail buffeting and dynamic instability, have been investigated extensively by Rockwell, D (1993) [76], Ozgoren, M. et al (2002) [77], Gordnier R. and Visbal M. (2005) [78], Menke M. et.al. (1997, 1999) [79, 80], Bisplinghoff, R. et al (1955) [46], Ashly H. et al (1967) [47], Gursul I. et al (1995) [81], Lowson M. (1988) [82], Rediniotis O. K. (1989) [83], Rediniotis O. et al (1990) [84], Traub L. (1995) [85], Moeller E. B. et al (2002) [86]. Oscillations “fluctuations” of vortex breakdown location have been investigated by Mitchell, A. et al (1998) [87], Menke et al (1997) [79], they performed flow visualization and velocity measurements to investigate the unsteady nature of vortex breakdown location and its oscillations over a delta wing as shown in figure 6.

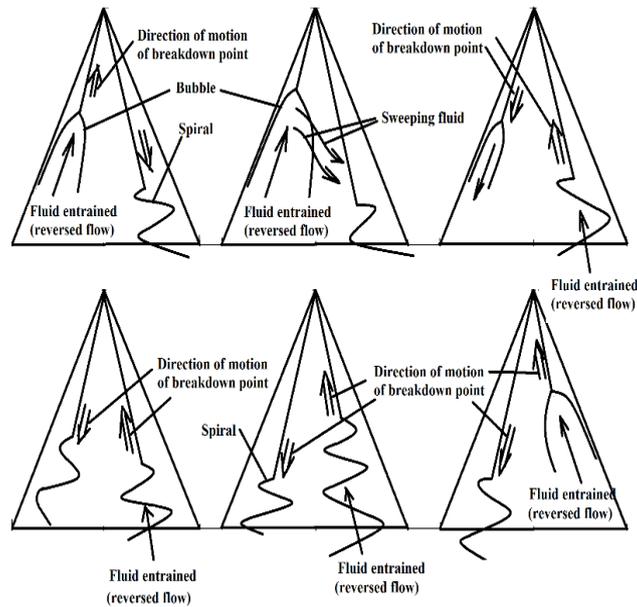


Figure 6: Altering vortex breakdown over delta wing.

The decreasing pitch of the vortex spiral at high angle of attack leads to an axisymmetric azimuthal vorticity distribution and consequently a stagnation point is formed at the viscous subcore initiating the bubble formation. Investigations on unsteady vortex flow included symmetric and antisymmetric vortex shedding by Gursul I. et al (1994) [4], Rediniotis O. K. et al (1990, 1993) [84, 88], Menke, M.; Yang, H.; Gursul, I. (1999) [80] and Stopountzis H. et al (1992) [89] as shown in figure 7, vortex wandering by Barnes C. et al (2015) [90] and Gursul I. et al (1995) [81]. Helical mode instability have been investigated by Gursul I. (2003) [91], and Gursul I. et al (1998) [92], shear layer instabilities by Garg A. and Leibovich S. (1979) [55], vortex asymmetry has been studied by Degani, D. (1991) [93], Degani, D.; Tobak M.(1992) [94]. The effect of Reynolds number on Asymmetric Vortex-Flow Pattern has been investigated by Stahl, W.H. and Asghar, A. (1996) [95]. More details in Omar, Salaheldin H. (2021) [96, 97,98].

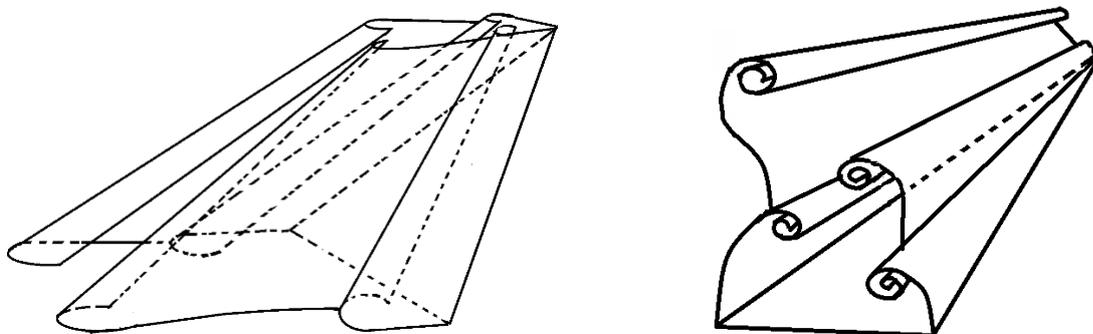


Figure 7: (a) symmetric vortex shedding

(b) antisymmetric vortex shedding

2.6 Factors affecting the vortex-bursting location of delta wings are;

2.6.1. Angle of attack and leading edge sweep: The location of the bursted vortex moves upstream with increasing the angle of attack and with decreasing the sweep angle, but there seems to be a final position of the breakdown location which lies for delta wings with sweep angles between 70° to 85° at about 30% to 40% of the wing chord Nangia R. K. 1989 [99].

2.6.2. Aspect ratio: An increase in aspect ratio or decrease in sweep angle results in upstream movement of the breakdown.

2.6.3. Probes: The effect of the presence of a probe, as multi hole probe or hotwire probe, in the vortex structure especially at the vortex core, on the vortex breakdown type or location, or on the structure pattern depends on the size of the probe, its position inside the vortex structure, the vortex strength, and on its position relative to the breakdown location of the vortex. Some wind tunnel experimental investigations on vortex breakdown over slender delta wings using hot wire probes or multi-hole probes are affected by the presence of the probe inside the vortex structure and reflect vortex breakdown at angles lower than those predicted using surface pressure measurements with no probe present in the vortex structure. It is to be expected that the presence of a hot wire probe or a multi-hole probe in the vortex structure a slender delta wing can result in dramatic change in the vortex structure or even in a vortex bursting.

2.6.4. Acceleration: During increasing the speed the breakdown moves upstream along the vortex and remained at a more forward position as long as the speed was increasing. As the speed approaches its new steady value the breakdown returns to its original location. The opposite occurs for decelerated flow Magness C. et al (1989) [100].

2.6.5. Suction: Breakdown can be eliminated by suction applied just downstream to the original bursting position Bushnell, D. M. (1992) [101], Dagenhart, J. et al (1981) [102], Polhamus, E. C. (1966) [103], Polhamus, E. C. (1971) [104]

2.6.6. Flaps: Effect of flap on the vortex flow over delta wings have been extensively investigated Rinoie, K. (1993) [105], Whitehead, A. H.; Keyes J. W. (1968) [106], Golubev, V. V. (1957) [107], Traub L. W. and Galls S. F. (1999) [110], Kuo C.H., and Hsu C. W. (1997) [111]. Upward deflection of a trailing edge flap leads to forward movement of the breakdown, while downward deflection leads to downstream movement of the breakdown Soltani, M. R. and Bragg, M. (1988) [112]. The effect of wing apex deflection on the surface pressure coefficient "Cp" distribution over a low-aspect-ratio highly swept arrow-wing configuration have been investigated Coe, Jr. P. L.; Weston R. P. (1979) [113]. The elastic deformation of the delta wing leading edge, trailing edge and the apex has an important influence on the flow field and on the vortex structure. A slight upward positive deflection of the wing apex leads to remarkable decrease in the pressure coefficients on the leeward side with inward displacement of the maximum suction pressure on the leeside of the wing as indication of the inward displacement of the primary vortex core attachment line of the flow around the primary vortex with the leeside surface and vice versa. Coe, Jr. P. L.; Weston R. P. (1979) [113], and the opposite is true.

2.6.7. Blowing: Blowing normal or parallel as well as directly down the vortex axis leads to

delaying the vortex breakdown. The energy of the blown air can be divided into two components, one parallel to the primary-vortex axis leading to an increase in the total head by increasing the axial velocity within the core and the other component normal to the primary vortex axis results in rolling up process and increasing the vortex strength Mitchell, A., et al (1998) [87], CUI, Y. D.; LIM, T. T.; TSAI H. M. (2007) [112], Bartasevicius Julius et al (2016) [113].

2.6.8. Wing profile: Wing profile as leading edge roundness, wing chamber affects the vortex formation, development and bursting. Nangia, R.; Hancock, G. (1970) [114].

2.6.9. Reynolds number effects: The flow are usually independent of Reynolds number especially for sharp edged delta wings Lambourne and Bryer (1961) [54]. The position of vortex breakdown over sharp edged delta wings does not have any strong Reynolds number effects because the separated free shear layer is fixed at the sharp leading edge. The suction peaks in the surface pressure coefficient "Cp" distributions of the leeward side of the delta wing, as indicators of the position of the axis of the primary and secondary vortices, are higher for turbulent flow. Increasing the Reynold number delays the secondary separation Kjelgaard, S. D.; Sellers, W. L.; Watson R. P. (1986) [115] and consequently the outboard displacement of the secondary vortex pushing the primary vortex outboard. Soltani and Bragg (1988) [110] stated that for sharp edged wings the Reynolds number effects are more obvious when the vortex breakdown location is approaching upstream to the trailing edge. Reynolds effect is more obvious for thicker profiles Wolffelt K.W. (1987) [116].

2.6.10. Denstiy Effect: Mohd-Zulhilmi, Paiz Ismadi (2011) [117] investigated experimentally the control of vortex breakdown by density effects inside a cylinder with a rotating top lid by injecting at the bottom a fluid with a small density difference. They stated that the injection of a heavy fluid creates a buoyancy force downward, which counteracts the meridional recirculation in the cylinder and thus enhances the formation of bubble-type vortex breakdown, while even a very small density difference of 0.02% is able to decrease by a factor of 2 the critical Reynolds number of appearance of the breakdown. This type of practice needs further similar investigations on all types of wings using gases instead of liquids to explore the possibility to control the vortex breakdown over wings.

REFERENCES

- [1] Philander, S. G. (1990). El Niño, La Niña, and the Southern Oscillation, Academic Press, Orlando, Florida, 289 pp.
- [2] D'Aleo, J. S. (2002). The Oryx Guide to El Niño and La Niña, Oryx Press, 230 pp.
- [3] Banea, S.P.M.; Zieglerb , J.L. (2015). Investigation of the Effect of Electrode Geometry on Spark Ignition, Combust. Flame, 162:462-469, 2015.[http:// dx.doi.org/10.1016/j.combustflame.2014.07.017](http://dx.doi.org/10.1016/j.combustflame.2014.07.017)
- [4] Gursul, I. (1994). Unsteady Flow Phenomena over Delta Wings at High Angle of Attack, AIAA Journal, Vol. 32, No. 2, February 1994, pp. 225-231.
- [5] Helmholtz, H. (1858). Über Integrale der hydrodynamischen Gleichungen, welche den

- Wirbelbewebungen entsprechen, J. Reine Angew. Math. 55 (1858) 25–55. Reprinted in: Wissenschaftliche Abhandlungen von Hermann Helmholtz (Barth, Leipzig, 1882) I 101–134.
- [6] Thorpe, S. A. 1968 A method of producing a shear-flow in a stratified fluid. J. Fluid Meeh. 32, 693-704.
- [7] Thorpe, S. A. (1971). Journal of Fluid Mechanics 46: 299{319 (1971).
- [8] Richardson Lewis Fry (1922). Weather Prediction by Numerical Process, Second Edition, Cambridge University Press, ISBN: 0-521-68044-1 (250pp)
- [9] Taylor, G. I., (1931). Effect of variation in density on the stability of superposed streams of fluid. Proc. R. Soc. London A, 132, 499-523.
- [10] Kundu, V.; Cohen I. M. (1971). Fluid Mechanics (Elsevier Academic Press, 3rd edition, 2004), 476-488.
- [11] Winant, C. D.; Browand, F. K. (1974). Vortex Pairing: The Mechanism of Turbulent Mixing-Layer Growth at Moderate Reynolds Number, in: Journal of Fluid Mechanics, pp 237-255, Vol. 63, Part 2, 1974.
- [12] Brown, G. L.; Roshko, A. (1974). On density effects and large structure in turbulent mixing layers. J. Fluid Meeh. 64, 775-816.
- [13] Winter, H. (1935). Strömungsvorgänge an platten und profilierten körpern bei kleinen spannweiten, Forschung auf dem Gebiet des Ingenieurwesens, 1935, vol. 6, no. 2, pp. 67-71.(In English)
- [14] Winter, H. (1936). Flow Phenomena on Plates and Airfoils of Short Span. NACA Rep. 798, 1936.
- [15] Wilson, H. A. Jr.; Lovell, J. C (1947). Full-Scale Investigation of the Maximum Lift and Flow Characteristics of an Airplane Having Approximately Triangular Plan Form. NACA RM No. L6K20, 1947.
- [16] Örnberg, T. A. (1954) Note on the Flow around Delta Wings. KTH AERO TN 38, 1954.
- [17] Werle, H. (1954). Quelques resultats experimentaux sur les ailes en fleches, aux faibles vitesses, obtenus en tunnel hydrodynamique," La Recherche Aeronautique 41, 1954.
- [18] Peckham, D. H.; Atkinson, S. A. (1957). Preliminary Results of Low Speed Wind Tunnel Tests on a Gothic Wing of Aspect Ratio 1.0, Report CP-508, Aeronautical Research Council, 1957.
- [19] Peckham, D. H. (1958). Low Speed Wind Tunnel Tests on a Series of Uncambered Slender Pointed Wings with Sharp Edges. R.A.E. Rep. No. AERO 2613, 1958.
- [20] Elle, B. J. (1958). An investigation at low speed of the flow near the apex of thin delta wings with sharp leading edges, Reports and Memoranda 3176, Aeronautical Research Council, January 1958.
- [21] Bergesen, A. J.; Porter J. D. (1960). An Investigation of the Flow aound Slender Delta Wings with Leading Edge Separation, M.Sc. Thesis in Engineering, Princeton University, 1960.
- [22] Weber, J. (1955). Some Effects of Flow Separation on Slender Delta Wings. R.A.E. TN 2425, 1955.
- [23] Marsden, D. J. (1957). Simpson, R. W. and Rainbird, W. J.; The Flow over Delta Wings at Low Speeds with Leading Edge Separation. College of Aeronautics, Cranfield, Rep. No. I 14, 1957.

- [24] Hummel, D. (1965). Untersuchungen über das Aufplatzen der Wirbel an schlanken Deltaflügeln. Zeitschrift für Flugwissenschaften, - 13, 5, pp. 158-168, 1965.
- [25] Hummel, D.; Srinivasan, P. S. (1967). Vortex Breakdown Effects on the Low-Speed Aerodynamic Characteristics of Slender Delta Wings in Symmetrical Flow, in J. Royal Aeronautical Society, Vol. 71, April, pp. 319-322.
- [26] Michalke, A. (1965). Vortex formation in a free boundary layer according to stability theory, J. Fluid Mech. (1965), vol. 22, part 2, pp. 371-383.
- [27] Stuart J. T. (1961). Adv. Aero. Sci. 3-4, 121-42, 1961.
- [28] Jones, J.P., (1960). The Breakdown of Vortices in Separated Flow. Dept. Aero. Astro. University of Southampton, Rep. No. 140.
- [29] Ludwieg, H., (1961). Contribution to the Explanation of the Instability of Vortex Cores above Lifting Delta Wings. Aero. Versuchsanstalt, Gottingen, Rep. AVA/61 A01.
- [30] Ludwieg, H. (1962). Zur Erklärung der Instabilität der über angestellten Deltaflügeln auftretenden freien Wirbelkerne, in: Zeitschrift für Flugwissenschaften, pp 242-249, Heft 6, 1962.
- [31] Ludwieg, H. (1965). Erklärung des Wirbelaufplatzens mit Hilfe der Stabilitätstheorie für Strömungen mit schraubenlinienförmigen Stromlinien, in: Zeitschrift für Flugwissenschaften, pp 437-442 Heft 12, 1965.
- [32] Squire, H. B. (1960). Analysis of the vortex breakdown, phenomenon. Part I, Aero. Dept., Imperial Coll., London, Rep. 102. Also in Miszellaneen der Angewandten Mechanik, pp. 306-12, Akademie-Verlag, Berlin.
- [33] Benjamin, T.B. (1962). Theory of the Vortex Breakdown Phenomenon, J. Fluid Mech., Volume 14, pp. 593-629.
- [34] Benjamin, T. B. (1967). Some developments in the theory of vortex breakdown. J. Fluid Mech., pp. 28, 65-84.
- [35] Sarpkaya, T. (1971). Vortex Breakdown in Swirling Conical Flows, in: AIAA Journal, pp 1792-1799, Vol. 9, No. 9, April 1971.
- [36] Piercy N.A.V. (1923). On the vortex pair quickly formed by some aerofoils, J. R. Aeronaut. Soc. 27 488-500.
- [37] Omar Salaheldin H., Adam Saad A. (2020) Wind Tunnel Investigation on Flow Visualization over Delta Wing at High Angle of Attack, International Journal of Core Engineering and Management, Volume 6, Issue 7, 2020 .
- [38] Earnshaw, P. B (1962). An experimental investigation of the structure of a leading edge vortex. ARC R & M 3281, 1962.
- [39] Payne, F. M.; Ng, T. T.; Nelson, R. C.; Schiff, L. B. (1986). Visualization and Flow Surveys of the Leading-Edge Vortex Structure on Delta Wing Planforms, AIAA Paper 86-0330, 1986.
- [40] Payne, F. M. (1987). The Structure of Leading Edge Vortex Flows Including Vortex Breakdown, Ph.D. thesis, Univ. of Notre Dame, Dept. of Aerospace and Mechanical Engineering, Notre Dame, IN, 1987.
- [41] Nelson, R.C.; Visser K. (1991). Breaking down the delta wing vortex: the role of vorticity in the breakdown process. AGARD, Vortex Flow Aerodynamics 15, 1991.

- [42] Riley A. J., Lawson M. V. (1998). Development of a three-dimensional free shear layer, *J. Fluid Mech.* 369, 49.
- [43] Delery, J. M. (1994) Aspects of Vortex Breakdown, *Progress in Aerospace Sciences*, Vol. 30, pp. 1-59, 1994.
- [44]Huang X. Z.; Hanff E. S. (1998). Flow Physics of Leading-Edge Vortex-Breakdown, *AIAA Paper* 98-31536.
- [45]Earnshaw, P. B.; Lawford, J. A. (1964). Low Speed Wind Tunnel Experiments on a Series of Sharp- edged Delta Wings, R & M 3424, Aeronautical Research Council, August 1964.
- [46] Bisplinghoff, R. L.; Ashley, H.; Halfman, R. L. (1955). *Aeroelasticity*. Addison-Wesley, Cambridge, Mass., 1955, pp. 418-419.
- [47] Ashly, H.; Bisplinghoff, R. (1967). *Aeroelasticity*, Cambridge, Mass., Addison Wesley, 1967.
- [48] Hall, M. G., (1966). "The structure of concentrated vortex cores," *Prog. Aero. Sci.* 7, pp. 53-110.
- [49] Mager, A. (1972). "Dissipation and breakdown of a wing-tip vortex," *J. Fluid Mech.* 55, pp. 609-28.
- [50] Leibovich S. (1978). The structure of vortex breakdown, *Annual Review Fluid Mech.* 10, 221 (1978).
- [51] Randall, J. D. and Leibovich, S., 1973. "The critical state: A trapped wave model of vortex breakdown," *J. Fluid Mech.* 58, pp. 495-515.
- [52] Harvey, J. K. (1962). Some observations of the vortex breakdown phenomenon. *J. Fluid Mech.* 14, 585-592.
- [53] Hall, M. G. A (1961). Theory for the core of a leading-edge vortex. *Journ. Fluid Mech.*, Vol. 11, 1961.
- [54] Lambourne, N. C.; Bryer, D. W. (1961). The Bursting of Leading-Edge Vortices - Some Observations and Discussions of the Phenomenon, Reports and Memoranda 3282, Aeronautical Research Council, 1961, Reports and Memoranda, Ministry of Aviation: London, United Kingdom, 1961, no. 3282.
- [55] Garg, A.K.; Leibovich, S. (1979). Spectral Characteristics of Vortex Breakdown Flowfields, *Physics of Fluids*, Vol. 22, No. 11, 1979, pp. 2053-2064. Unsteady Aspects of Leading-edge Vortices RTO-TR-AVT-080 6 -33
- [56] Schade, H.; Michalke, A. (1962) Zur Entstehung von Wirbeln in einer freien Grenzschicht, in: *Zeitschrift für Flugwissenschaften*, Heft 4/5, pp 147-154, Oktober 1962.
- [57] Hall, M.G. (1972). Vortex breakdown. *Ann. Rev. Fluid Mech.* 4, 195-218 (1972)
- [58] Reynolds, W.C.; Carr L. W. (1985). Review of Unsteady Driven Separated Flows. *AIAA paper* 85-0527, 1985.
- [59] Lee, M.; Ho, C. M. (1989). Vortex Dynamics of Delta Wings, *Frontiers In Exper. Fluid Mech., Lecture Notes In Eng.*, pp 365-427 Vol. 46, Springer Verlag, Berlin 1989
- [60] Elle, B. J. (1958). An investigation at low speed of the flow near the apex of thin delta wings with sharp leading edges, Reports and Memoranda 3176, Aeronautical Research Council, January 1958.
- [61] Lee, M.; Ho C. M. (1990). Lift force of delta wings, *Appl. Mech. Rev.* 43, 209 (1990).

- [62] Erickson, G. E. (1982). Water-Tunnel Studies of Leading-Edge Vortices, *Journal of Aircraft*, Vol. 19, No. 6, 1982, pp. 442-448.
- [63] Lee, G. H. (1955). Notes on the flow around delta wings with sharp leading edges. *ARC R & M* 3070, 1955.
- [64] Visser K. D.; Nelson R. C. (1993) Measurements of Circulation and Vorticity in the Leading-Edge Vortex of a Delta Wing, *AIAA Journal* Vol. 31, No 1, January 1993.
- [65] Visser, K.; Ferrero, M.; Nelson R. (2004). Physical Considerations of Leading Edge Flows. 22nd Applied Aerodynamics Conference and Exhibit, 2004.
- [66] Wang, J.; Zhan J. (2005). New pair of leading-edge vortex structure for flow over delta wing, *Journal of Aircraft*, vol. 42, no. 3, pp. 718-721, 2005.
- [67] Lowson, M. V.; (1964). Some experiments with vortex breakdown, *J. Royal Aeronaut. Soc.* 68, 343 (1964).
- [68] Greenwell, D.I; Wood, N.J. (1994). Some Observations on the Dynamic Response to Wing Motion of the Vortex Burst Phenomenon, *Aeronautical Journal*, February 1994, pp. 49-59.
- [69] Wentz, W. H.; Kohlman, D. L. (1971). Vortex breakdown on slender sharp-edge wings, *Journal of Aircraft*, Vol. 8, No. 3, pp. 156-161, 1971.
- [70] Wood, R. M.; Miller, D. S. (1985) Fundamental Aerodynamic Characteristics of Delta Wings with Leading-Edge Vortex Flows, in: *Journal of Aircraft*, pp 479-485, Vol. 22, No. 6, June 1985.
- [71] Wentz, W. H. Jr. (1968). Wind Tunnel Investigations of Vortex Breakdown on Slender Sharp-Edged Wings, Ph.D. Thesis, Univ. of Kansas, Lawrence, Kansas, 1968.
- [72] Hensch, M. J.; Luckring, J. M. (1990) Connection between Leading-Edge Sweep, Vortex Lift, and Vortex Strength for Delta Wings," *Journal of Aircraft*, Vol. 27, 1990, pp. 473-475.
- [73] Kegelmann, J.; Roos, F. (1989). Effects of Leading-Edge Shape and Vortex Burst on the Flow Field of a 70 Degree Sweep Delta-Wing, *AIAA Paper* 89-0086, 1989.
- [74] Faler, J. H.; Leibovich, S. (1977). Disrupted states of vortex flows and vortex breakdown. *Phys. Fluids* 20, 1385-1400 (1977).
- [75] Faler, J. H.; Leibovich, S. (1978). An experimental map of the internal structure of a vortex breakdown, *J. Fluid Mech.* 86, 313 (1978).
- [76] Rockwell, D. (1993). Three-Dimensional Flow Structure on Delta Wings at High Angle-of-Attack: Experimental Concepts and Issues, *AIAA Paper* 93-0550, January, 1993.
- [77] Ozgoren, M.; Shahin, B.; Rockwell, D. (2002) Vortex Structure on a Delta Wing at High Angle of Attack, *AIAA Journal*, Vol. 40, No. 2, February 2002, pp. 285-292.
- [78] Gordnier, R.; Visbal, M. (2005). Unsteady aerodynamics of nonslender delta wings, *Progress in Aerospace Sciences* 41, pp.515-557 2005
- [79] Menke, M.; Gursul, I. (1997). Unsteady nature of leading edge vortices," *Physics of Fluids*, Vol. 9, pp 2960.
- [80] Menke, M.; Yang, H.; Gursul, I. (1999). Experiments on the unsteady nature of vortex breakdown over delta wings, *Experiments in Fluids*, Vol. 27, No. 3, pp. 262-272, 1999.
- [81] Gursul, Ismet; Yang Houbin (1995). On fluctuations of vortex breakdown location, *Physics of Fluids* 7, 229.

- [82] Lawson, M. V. (1988). The Three Dimensional Vortex Sheet Structure on Delta Wings, AGARD CP-438, Fluid Dynamics of Three-Dimensional Turbulent Shear Flows and Transition, 1988.
- [83] Rediniotis, O. K.; Telionis, D. P. (1989). Periodic Vortex Shedding over Delta Wings, in: AIAA-89-1923, June.
- [84] Rediniotis, O.K.; Stapountzis, H.; Telionis, D.P. (1990). Vortex Shedding over Delta Wings, AIAA Journal, Vol. 28, No. 5, 1990, pp. 944-946.
- [85] Traub, L. W. (1995). Aerodynamic characteristics of vortex flaps on a double-delta planform, Journal of Aircraft 89:1995, vol. 32, no. 2, pp. 449-450.
- [86] Moeller, E. B.; Rediniotis, O. K. (2002). Hingeless Flow Control over a Delta-Wing Platform, Journal of Aircraft 39:6, 1035-1044, 2002.
- [87] Mitchell, A.; Barberis, D.; Delery, J. (1998). Oscillation of Vortex Breakdown Location and Its Control by Tangential Blowing, AIAA 98-2914, Albuquerque, June 1998
- [88] Rediniotis, O.K.; Stapountzis, H.; Telionis, D.P. (1993). Periodic Vortex Shedding over Delta Wings, AIAA Journal, Vol. 31, 1993, pp. 1555-1562.
- [89] Stopountzis, H.; Iliadis G.; Agelakis J.; Mansour G. (1992). Vortex Shedding Over Delta Wings and Plates at High Angle Of Incidence, pp 331-341, ZFW 16, 1992
- [90] Barnes, C. J.; Visbal M. R.; Gordnier R. E. (2015). Analysis of streamwise-oriented vortex interactions for two wings in close proximity; Physics of Fluids 27, 015103, 2015)
- [91] Gursul, I. (2003). Review of unsteady vortex flows over delta wings, AIAA Paper 2003-3942, 2003.
- [92] Gursul, I.; Xie, W. (1998). Physics of Buffeting Flows over Delta Wings, AIAA 98-0688, 36 th Aerospace Sciences Meeting and Exhibit, January 12-15, 1998, Reno, NV.
- [93] Degani, D. (1991). Effect of Geometrical Disturbance on Vortex Asymmetry, AIAA Journal, Vol. 29, No. 4, pp. 560-566, 1991., Instabilities of Flows over Bodies at Large Incidence, AIAA Journal, Vol. 30, No. 1, pp. 94-100, 1992.
- [94] Degani, D.; Tobak M. (1992). Experimental Study of Controlled Tip Disturbance Effect on Flow Asymmetry, Physics of Fluids A, Vol. 4, No. 12, pp. 2825- 2832, 1992.
- [95] Stahl W.H.; Asghar A. (1996). Change of Asymmetric Vortex-Flow Pattern as Function of Reynolds Number and Incidence behind Circular Cone, AIAA Paper 96-3389, 1996.
- [96] Omar, Salaheldin H. (2021) A Review on Unsteady Vortex Flow over Delta Wings at High Angles of Attack, under publication.
- [97] Omar, Salaheldin H. (2021) Unsteady Surface Pressure Measurements in Wind tunnel at different (angles of attack), under publications.
- [98] Omar, Salaheldin H. (2021) Unsteady Surface Pressure Measurements at different sweep Angles and angles of Attacks in Wind Tunnel), under publication.
- [99] Nangia, R. K. (1989). Development of an Attained. LE Thrust Method for Use in Combat Aircraft Wing Design. Contractor Report, 1989.
- [100] Magness, C.; Robinson, O.; Rockwell D. (1989). Control of Leading Edge Vortices on a Delta Wing, AIAA paper 89-0999, 1989.
- [101] Bushnell, D. M. (1992). Longitudinal vortex control-techniques and applications, in: Aerounautical Journal, October 1992.

- [102] Dagenhart, J. R. (1981). Amplified Cross flow Disturbances in the Laminar Boundary Layer on Swept Wings with Suction, in: NASA Technical Paper 1902, November 1981.
- [103] Polhamus, E. C. (1966). A Concept In The Vortex Lift Of Sharp-Edge Delta Wings Based On Leading Edge Suction Analogy, NASA TN D- 3767, 1966
- [104] Polhamus, E. C. (1971); Predictions of vortex-lift characteristics by a leading-edge suction analogy. *Journ.of Aircraft*, Vol. 8, No. 4, April. 1971.
- [105] Rinoie, K. (1993). Experiments on a 60 delta wing with vortex flaps and vortex plates, *Aeronautical Journal*, January 1993.
- [106] Whitehead, A. H.; Keyes J. W. (1968). Flow phenomena and separation over delta wings with trailing-edge flaps at Mach 6. *AIAA Journ.* vol. 6, No. 12, pp, 2380, Dec. 1968.
- [107] Golubev, V. V. (1957). On the influence of flaps on the lift force of a wing, *Trudy Tsentr. Aero Gidrodin. Inst.* (1938) No 342 24–35 (in Russian). Reprinted in: *Trudy po aerodinamike* (Papers on Aerodynamics) (GITTL, Moscow-Leningrad, 1957) 671–687.
- [108] Traub, L. W.; Galls S. F. (1999). Effects of leading- and trailing-edge Gurney flaps on a delta wing, *Journal of Aircraft*, 1999, vol. 36, no. 4, pp. 651-658.
- [109] Kuo, C. H.; Hsu C. W. (1997). Development of vortex structure over delta wing with leading-edge flap, *Journal of Aircraft*, 1997, vol. 34, no. 5, pp. 577-584.
- [110] Soltani, M. R.; Bragg M. B. (1988) Experimental Measurements on an Oscillating 70-degree Delta Wing in Subsonic Flow, *AIAA Paper 88-2576-CP*, 1988.
- [111] Coe, Jr. P. L.; Weston R. P. (1979). Effects of Wing Leading-Edge Deflection on Low-Speed Aerodynamic Characteristics of a Low-Aspect-Ratio Highly Swept Arrow-Wing Configuration, in: *NASA Technical Paper 1434*, June 1979.
- [112] CUI, Y. D.; LIM, T. T.; TSAI H. M. (2007). Control of Vortex Breakdown over Delta Wing Using Freebody Slot Blowing, *AIAA Journal*, Vol 45, No.1, 2007. , Freebody Slot Blowing on Vortex Breakdown and Load over a Delta Wing, *AIAA Journal*, Vol 46, No.3, 2008.
- [113] Bartasevicius, Julius; Buzica, Andrei; (2016). Breitsamter Christian; Discrete vortices on delta wings with unsteady leading-edge blowing, 8th AIAA Flow Control Conference, 2016.
- [114] Nangia, R.; Hancock, G. (1970). Delta wings with longitudinal camber at low speed, Queen Mary college, University of London, United Kindom, 1970.
- [115] Kjelgaard, S. D.; Sellers, W. L.; Watson R. P. (1986). The Flow Field over a 75 Degree Swept Delta Wing at 20.5 Degrees Angle of Attack, *AIAA Paper 86-1775*, 1986.
- [116] Wolffelt, K.W. (1987). Investigation on the Movement of Vortex Burst Position with Dynamically Changing Angle of Attack for a Schematic Delta Wing in a Water Tunnel with Correlation to Similar Studies in Wind Tunnel, *Aerodynamic and Related Hydrodynamic Studies Using Water Facilities*, AGARD-CP-413, 1987.
- [117] Mohd-Zulhildi, Paiz Ismadi (2011). Patrice, Meunier, Andreas. Fouras, and Kerry Hourigan; Experimental control of vortex breakdown by density effects, *Physics of Fluids* 23, 034104, 2011.
- [118] Omar Salaheldin H. (2020); Wind Tunnel Investigation on Surface Pressures and Vortex Flow over Delta Wing at High Angles of Attack and Free-Stream Velocities, *International Journal of Core Engineering and Management*, Volume 6 , Issue 6, 2020.