

**DESIGN OF A GAIN SCHEDULING CONTROLLER FOR THREE-TANK SYSTEM
(3TS)**

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Abstract

This paper presents the design of a gain-scheduling controller for a nonlinear three-tank system (3TS). The system is linearized at different operating points, and proportional-integral (PI) controllers are designed using Cohen-Coon, Ziegler-Nichols, and Chien-Hrones-Reswick tuning rules. Cohen-Coon's tuning rule, with the lowest errors, was selected. MATLAB/Simulink was used to implement the gain-scheduling mechanism and evaluate controller performance. Results show satisfactory performance at certain input ranges but poor tracking in others, highlighting areas for further tuning.

Index Terms – Gain scheduling, three-tank system, nonlinear system, PI controller, linearization, MATLAB/Simulink, control system, tuning methods.

I. INTRODUCTION

The project background highlights the principle of feedback control, which aims to maintain consistent system performance by adjusting controller parameters to address uncertainties or changing setpoints. Most traditional controllers use fixed parameters, which are insufficient for systems characterized by varying or uncertain parameters, such as an aircraft's mass reduction during flight due to fuel consumption. Adaptive control addresses this issue by combining parameter estimation with control law adjustments to adapt to changing conditions in real-time[1]. Unlike robust control, adaptive control does not require prior information on parameter bounds, making it more suitable for slow or unpredictable variations.

Gain scheduling is a type of adaptive control used for systems with predictable parameter changes. This project aims to design gain-scheduled proportional-integral-derivative (PID) controllers for a laboratory-scale three-tank system, using the internal model control (IMC) technique. The designed controllers are compared with those implemented using fixed-parameter techniques and other gain-scheduled designs. The project addresses the gap in designing gain-scheduled controllers for multi-input, multi-output (MIMO) systems, with a focus on their application to the three-tank system. MATLAB/Simulink is used for modelling, controller design,

and evaluation of the control performance.

II. LITERATURE REVIEW

The literature review focuses on adaptive control techniques, specifically gain scheduling, as a means of maintaining control performance when parameters are unknown or vary over time. Adaptive control is essential for maintaining desired system behaviour in dynamic environments, such as those encountered in aerospace and industrial processes. Gain scheduling, a popular adaptive control method, is used to adjust controller parameters according to the system's operating conditions, thereby effectively compensating for parameter variations[1].

Several studies have explored adaptive control and its applications across different domains. Miao Wang and Francesco Crusca implemented a gain scheduling controller for water level control, showing improved performance over fixed controllers [2]. Similarly, Qiang Liu developed a gain scheduling adaptive control for tension control in continuous annealing processes, resulting in enhanced stability [3]. Jinho Jang and others proposed an adaptive control technique for nonlinear systems with parametric uncertainties, demonstrating its effectiveness in a high-performance aircraft model [4]. These studies underline the versatility of gain scheduling in dealing with changing operating points.

S.M. Sharhruz and S. Behtash designed controllers for linear parameter-varying systems using gain scheduling, effectively placing the closed-loop poles in desired locations [5]. The gain scheduling technique has also been applied in complex systems such as overhead cranes and bioreactors [6] [7] to maintain stability and control precision in response to parameter variations. In particular, Zavari Keivan and others demonstrated the use of multi-objective H_{∞} methods to balance reference tracking and disturbance rejection for overhead cranes [6].

The success of gain scheduling in adaptive control is attributed to its ability to respond to predictable variations in system parameters. This ability makes it an attractive method for industrial control systems, such as those encountered in chemical process control, nonlinear robotics and mechatronic systems [8][9].

Naik presented an adaptive PI controller for conical tank processes using gain scheduling, achieving faster response times compared to conventional PI controllers [10]. This study illustrates how gain scheduling can improve process control even in relatively simple systems. The method was further validated in the design of a gain-scheduled controller for gas turbine engines, ensuring stability and tracking performance across a wide range of operating conditions [11].

The literature demonstrates that gain scheduling is widely applicable in adaptive control for a variety of industries, from aerospace and robotics to industrial manufacturing. The main advantage lies in its ability to change the controller parameters swiftly to respond to operating conditions, effectively compensating for parameter uncertainties. However, despite its effectiveness, gain scheduling does not always guarantee stability under rapid parameter changes, which presents an opportunity for further research and optimization.

- Design of Gain-Scheduled Controllers: Gain scheduling involves creating controllers for different LTI models by adjusting the parameters of the controllers based on the system's operating conditions. The linear models obtained for the three-tank system were used to design proportional-integral (PI) controllers. Three different tuning rules (Cohen-Coon, Ziegler-Nichols, and Chien-Hrones-Reswick) were used to develop the controllers. The Integral Absolute Error (IAE) and Integral Square Error (ISE) were calculated to evaluate the performance of each controller. The controllers are represented Eq. 1, Eq. 2 and Eq. 3

$$K(s,p) = \frac{1}{s} \begin{bmatrix} 78.6 & 67.1 \\ 3.8 & 5.1 \end{bmatrix} \quad \text{Eq.1}$$

$$K(s,p) = \frac{1}{s} \begin{bmatrix} 36 & 0 \\ 1.7 & 0.47 \end{bmatrix} \quad \text{Eq. 2}$$

$$K(s,p) = \frac{1}{s} \begin{bmatrix} 143.7(1 + 0.3s) & 31.7(1 + 0.47s) \\ 6.1(1 + 0.56s) & 5.5 \end{bmatrix} \quad \text{Eq. 3}$$

B. Implementation in Simulink

Matlab/Simulink was used to implement the gain scheduling mechanism by creating look-up tables that relate operating points to the corresponding gains. This ensures that controller parameters change quickly in response to changes in system dynamics. The look-up tables were created to contain the proportional and integral gains of the two PI controllers for seven different operating points, ensuring efficient adaptation to changes in the three-tank system's operating conditions.

C. Evaluation of Controller Performance

The gain scheduler controllers were evaluated for their ability to maintain desired water levels in Tanks 1 and 2. The simulation results showed that the gain-scheduled controllers effectively adjusted their parameters to accommodate changes in operating conditions. However, in specific scenarios (such as step changes from 16.800 to 15.145), the controllers required further tuning for improved performance.

IV. IMPLEMENTATION AND RESULTS

The three-tank system (3TS) serves as a benchmark for demonstrating control concepts, such as linear, nonlinear control, and fault detection. This section describes the nonlinear model construction in Simulink, its linearization, and controller implementation. Different techniques are used, including the use of look-up tables to manage system complexities and dynamic behaviours.

A. Mathematical Preliminaries

The system is a two-input, two-output process where the controlled variables are the water levels in Tanks 1 and 2, and the manipulated variables are the flow rates of two pumps. Tank 3's water level is observed but not controlled. The nonlinear equations of the three-tank system is given

by Eq. 4, Eq. 5 and Eq. 6.

$$\frac{dh_1}{dt} = -\left(\frac{1}{S}\right) \cdot \mu_1 \cdot Sp \cdot \sqrt{2g} \cdot \text{sgn}(h_1 - h_3) \cdot \sqrt{|h_1 - h_3|} + \left(\frac{q_1}{S}\right) - \left(\frac{d_1}{S}\right) \quad \text{Eq. 4}$$

$$\frac{dh_2}{dt} = \left(\frac{1}{S}\right) \cdot \mu_3 \cdot Sp \cdot \sqrt{2g} \cdot \text{sgn}(h_3 - h_2) \cdot \sqrt{|h_3 - h_2|} - \left(\frac{1}{S}\right) \cdot \mu_2 \cdot Sp \cdot \sqrt{2g} \cdot \text{sgn}(h_2) \cdot \sqrt{|h_2|} + \left(\frac{q_2}{S}\right) - \left(\frac{d_2}{S}\right) \quad \text{Eq. 5}$$

$$\frac{dh_3}{dt} = \left(\frac{1}{S}\right) \cdot \mu_1 \cdot Sp \cdot \sqrt{2g} \cdot \text{sgn}(h_1 - h_3) \cdot \sqrt{|h_1 - h_3|} - \left(\frac{1}{S}\right) \cdot \mu_3 \cdot Sp \cdot \sqrt{2g} \cdot \text{sgn}(h_3 - h_2) \cdot \sqrt{|h_3 - h_2|} - \left(\frac{d_3}{S}\right) \quad \text{Eq. 6}$$

B. Computation of Steady state controlled variable values

The nonlinear equations of Eq. 4, Eq. 5 and Eq. 6 are implemented in SIMULINK, with nominal parameter values for $S, \mu_1, \mu_2, \mu_3, Sp$ and nominal operating point values for h_1, h_2, h_3, q_1, q_2 , with disturbances eventually simulated by introducing leaks to tank 3 only i.e. $d_1 = d_2 = 0$ and the nominal operating point values for h_1, h_2, h_3 found by running the constructed diagram with nominal manipulated input values of:

$$q_{10} - q_{16} = (32, 34, 36, 38, 40, 42, 44) \text{ cm}^3/\text{s} \quad \text{Eq. 7}$$

$$q_{20} - q_{26} = (43, 45, 47, 49, 51, 53, 55) \text{ cm}^3/\text{s} \quad \text{Eq. 8}$$

The following are the steady state values obtained for h_1, h_2 , and h_3 using Simulink.

$$h_{10} - h_{16} = [37.749, 42.310, 47.128, 52.199, 57.523, 63.097, 68.918] \quad \text{Eq. 9}$$

$$h_{20} - h_{26} = [15.145, 16.800, 18.540, 20.363, 22.270, 24.259, 26.329] \quad \text{Eq. 10}$$

$$h_{30} - h_{36} = [26.970, 30.145, 33.493, 37.013, 40.704, 44.564, 48.590]. \quad \text{Eq. 11}$$

C. Linearization of the Nonlinear State Equations

The nonlinear model is linearized around nominal operating conditions using MATLAB's "linmod" command. This generates a linear, state-space representation for different operating points. Eq. 12 to Eq. 18 shows seven different linear models are developed, forming the basis for PI controller design.

The tuning rule with the lowest errors is adopted for controller design across all operating points and transfer functions for each model are derived using the (ss2tf) command as shown in Eqns. Three tuning rules – Ziegler-Nichols, Cohen-Coon, and Chien-Hrones-Reswick – are evaluated using Integral Absolute Error (IAE) and Integral Square Error (ISE) as shown in Table 1.

Table 1: The three different tuning rules and their corresponding IAE and ISE values.

Type of tuning rule	IAE1	IAE2	ISE1	ISE2
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Cohen Coon	26.5	108.1	7.783	21.1
Zeigler Nichols	28.45	245	8.228	47.26
Chien-Hrones-Reswick	42.77	399.2	12.5	92.33

$$P(s) = \frac{\frac{0.006711 s^2 + 0.0003002 s + 2.73e-06}{s^3 + 0.05469 s^2 + 0.000753 s + 1.502e-06}}{\frac{6.068e-07}{s^3 + 0.05469 s^2 + 0.000753 s + 1.502e-06}} \quad \frac{\frac{6.068e-07}{s^3 + 0.05469 s^2 + 0.000753 s + 1.502e-06}}{\frac{0.006711 s^2 + 0.0001946 s + 6.068e-07}{s^3 + 0.05469 s^2 + 0.000753 s + 1.502e-06}} \quad \text{Eq. 12}$$

$$Q(s) = \frac{\frac{0.006711 s^2 + 0.0002835 s + 2.435 \times 10^{-6}}{s^3 + 0.05162 s^2 + 0.0006709 s + 1.264 \times 10^{-6}}}{\frac{5.377 \times 10^{-7}}{s^3 + 0.05162 s^2 + 0.0006709 s + 1.264 \times 10^{-6}}} \quad \frac{\frac{5.377 \times 10^{-7}}{s^3 + 0.05162 s^2 + 0.0006709 s + 1.264 \times 10^{-6}}}{\frac{0.006711 s^2 + 0.0001832 s + 5.377 \times 10^{-7}}{s^3 + 0.05162 s^2 + 0.0006709 s + 1.264 \times 10^{-6}}} \quad \text{Eq. 13}$$

At operating point H1,H2 and H3=[42.310,16.800,30.145] respectively.

$$R(s) = \frac{\frac{0.006711 s^2 + 0.0002686 s + 2.186 \times 10^{-6}}{s^3 + 0.04887 s^2 + 0.0006016 s + 1.073 \times 10^{-6}}}{\frac{4.798 \times 10^{-7}}{s^3 + 0.04887 s^2 + 0.0006016 s + 1.073 \times 10^{-6}}} \quad \frac{\frac{4.798 \times 10^{-7}}{s^3 + 0.04887 s^2 + 0.0006016 s + 1.073 \times 10^{-6}}}{\frac{0.006711 s^2 + 0.000173 s + 4.798 \times 10^{-7}}{s^3 + 0.04887 s^2 + 0.0006016 s + 1.073 \times 10^{-6}}} \quad \text{Eq. 14}$$

At operating point H1,H2 and H3=[47.128,18.540,33.493] respectively

$$S(s) = \frac{\frac{0.006711 s^2 + 0.0002552 s + 1.973 \times 10^{-6}}{s^3 + 0.04641 s^2 + 0.0005427 s + 9.198 \times 10^{-7}}}{\frac{4.308 \times 10^{-7}}{s^3 + 0.04641 s^2 + 0.0005427 s + 9.198 \times 10^{-7}}} \quad \frac{\frac{4.308 \times 10^{-7}}{s^3 + 0.04641 s^2 + 0.0005427 s + 9.198 \times 10^{-7}}}{\frac{0.006711 s^2 + 0.000164 s + 4.308 \times 10^{-7}}{s^3 + 0.04641 s^2 + 0.0005427 s + 9.198 \times 10^{-7}}} \quad \text{Eq. 15}$$

At operating point H1,H2 and H3=[52.199,20.363,37.013] respectively.

$$T(s) = \frac{\frac{0.006711 s^2 + 0.0002431 s + 1.791 \times 10^{-6}}{s^3 + 0.04419 s^2 + 0.000492 s + 7.943 \times 10^{-7}}}{\frac{3.891 \times 10^{-7}}{s^3 + 0.04419 s^2 + 0.000492 s + 7.943 \times 10^{-7}}} \quad \frac{\frac{3.891 \times 10^{-7}}{s^3 + 0.04419 s^2 + 0.000492 s + 7.943 \times 10^{-7}}}{\frac{0.006711 s^2 + 0.0001558 s + 3.891 \times 10^{-7}}{s^3 + 0.04419 s^2 + 0.000492 s + 7.943 \times 10^{-7}}} \quad \text{Eq. 16}$$

At operating points H1,H2 and H3=[57.523,22.270,40.704] respectively

$$U(s) = \frac{\frac{0.006711 s^2 + 0.0002321 s + 1.633 \times 10^{-6}}{s^3 + 0.04217 s^2 + 0.0004482 s + 6.907 \times 10^{-7}}}{\frac{3.531 \times 10^{-7}}{s^3 + 0.04217 s^2 + 0.0004482 s + 6.907 \times 10^{-7}}} \quad \frac{\frac{3.531 \times 10^{-7}}{s^3 + 0.04217 s^2 + 0.0004482 s + 6.907 \times 10^{-7}}}{\frac{0.006711 s^2 + 0.0001484 s + 3.531 \times 10^{-7}}{s^3 + 0.04217 s^2 + 0.0004482 s + 6.907 \times 10^{-7}}} \quad \text{Eq. 17}$$

At operating points H1,H2 and H3=[63.097,24.259,44.564] respectively.

$$V(s) = \frac{\frac{0.006711 s^2 + 0.000222 s + 1.495 \times 10^{-6}}{s^3 + 0.04034 s^2 + 0.0004101 s + 6.046 \times 10^{-7}}}{\frac{3.22 \times 10^{-7}}{s^3 + 0.04034 s^2 + 0.0004101 s + 6.046 \times 10^{-7}}} \quad \frac{\frac{3.22 \times 10^{-7}}{s^3 + 0.04034 s^2 + 0.0004101 s + 6.046 \times 10^{-7}}}{\frac{0.006711 s^2 + 0.0001418 s + 3.22 \times 10^{-7}}{s^3 + 0.04034 s^2 + 0.0004101 s + 6.046 \times 10^{-7}}} \quad \text{Eq. 18}$$

Cohen Coon tuning rule was used to design 2*49 PI Controllers. The P(Proportional) gains for the first controller are shown in the table below. The table shows the corresponding operating points H1 and H2 values with the corresponding proportional gain.

Table 2: The Proportional gains for the first PI controller with highlighted operating points H1 and H2 values.

H2 \ H1	15.145	16.800	18.540	20.363	22.270	24.259	26.329
37.749	8.31503	7.89785	7.52398	7.20297	6.90291	6.66570	6.43614
42.310	8.35905	7.90065	7.52265	7.17426	6.88190	6.61301	6.37309
47.128	8.39956	7.92702	7.51126	7.15961	6.86951	6.60160	6.34310
52.199	8.47564	7.98013	7.55282	7.20075	6.84914	6.58197	6.32018
57.523	8.53446	8.03059	7.60135	7.22386	6.86418	6.58323	6.30042
63.097	8.63416	8.11295	7.65372	7.24972	6.87445	6.57802	6.31433
68.918	8.74630	8.19111	7.70074	7.30347	6.91929	6.60690	6.31035

Table 3: The Integral gains of the first PI controller with highlighted operating point H1 and H2 values

H2 \ H1	15.145	16.800	18.540	20.363	22.270	24.259	26.329
37.749	0.123262004	0.109963345	0.098715369	0.089527875	0.081414991	0.075207093	0.069453030
42.310	0.126131377	0.111394424	0.099938405	0.089973112	0.081993021	0.074946564	0.068989336
47.128	0.128778694	0.113488767	0.100800358	0.090637798	0.082657379	0.075629249	0.069185460
52.199	0.132521716	0.116264908	0.103066537	0.092773022	0.083059520	0.076025044	0.069460674
57.523	0.135664629	0.118869596	0.105459609	0.094311826	0.084342925	0.076865793	0.069767700
63.097	0.140148821	0.122491752	0.107946536	0.095893496	0.085406104	0.077482497	0.070824682
68.918	0.145067006	0.125958794	0.110230702	0.098236783	0.087350500	0.0789326343	0.071363748

Table 4: The Proportional gains of the second PI controller with highlighted operating points H1 and H2 values.

H2 \ H1	15.145	16.800	18.540	20.363	22.270	24.259	26.329
H1							

37.749	6.80847	6.48958	6.20182	5.93653	5.69604	5.47241	5.26696
42.310	6.77881	6.46223	6.17651	5.91345	5.67127	5.45288	5.24795
47.128	6.74784	6.43583	6.14862	5.88813	5.65142	5.42995	5.22643
52.199	6.72011	6.40704	6.12502	5.86557	5.63028	5.41012	5.20803
57.523	6.69232	6.38169	6.10151	5.84291	5.60576	5.39099	5.19058
63.097	6.66447	6.35625	6.07659	5.82085	5.57534	5.37162	5.17201
68.918	6.63840	6.33162	6.05495	5.79927	5.56652	5.35236	5.15447

Table 5: The Integral gains of the second PI controller with highlighted operating points H1 and H2 values

H2 \ H1	15.145	16.800	18.540	20.363	22.270	24.259	26.329
37.749	0.025284342	0.023063448	0.021130282	0.019434655	0.017926406	0.016617574	0.015434720
42.310	0.024928124	0.022761783	0.020859000	0.019200853	0.017728787	0.016423420	0.015255580
47.128	0.024603906	0.022459051	0.020595540	0.018963959	0.017510585	0.016239677	0.015086612
52.199	0.024291584	0.022198022	0.020356442	0.018755277	0.017307987	0.016052908	0.014921995
57.523	0.023970685	0.021906299	0.020109356	0.018523543	0.017121168	0.015869187	0.014765642
63.097	0.023678795	0.021649705	0.018315770	0.018315770	0.016859605	0.015696249	0.014616255
68.918	0.023388543	0.021388718	0.018104799	0.018104799	0.016750104	0.015539934	0.014457566

D. Gain Scheduling Mechanism

The Gain scheduling mechanism in Figure 2 is implemented using four 2D look-up tables in Simulink, which link operating points H1 and H2 with proportional and integral gains. The look-up tables ensure quick adaptation of controller parameters in response to changes in operating

conditions, enhancing system performance under varying conditions.

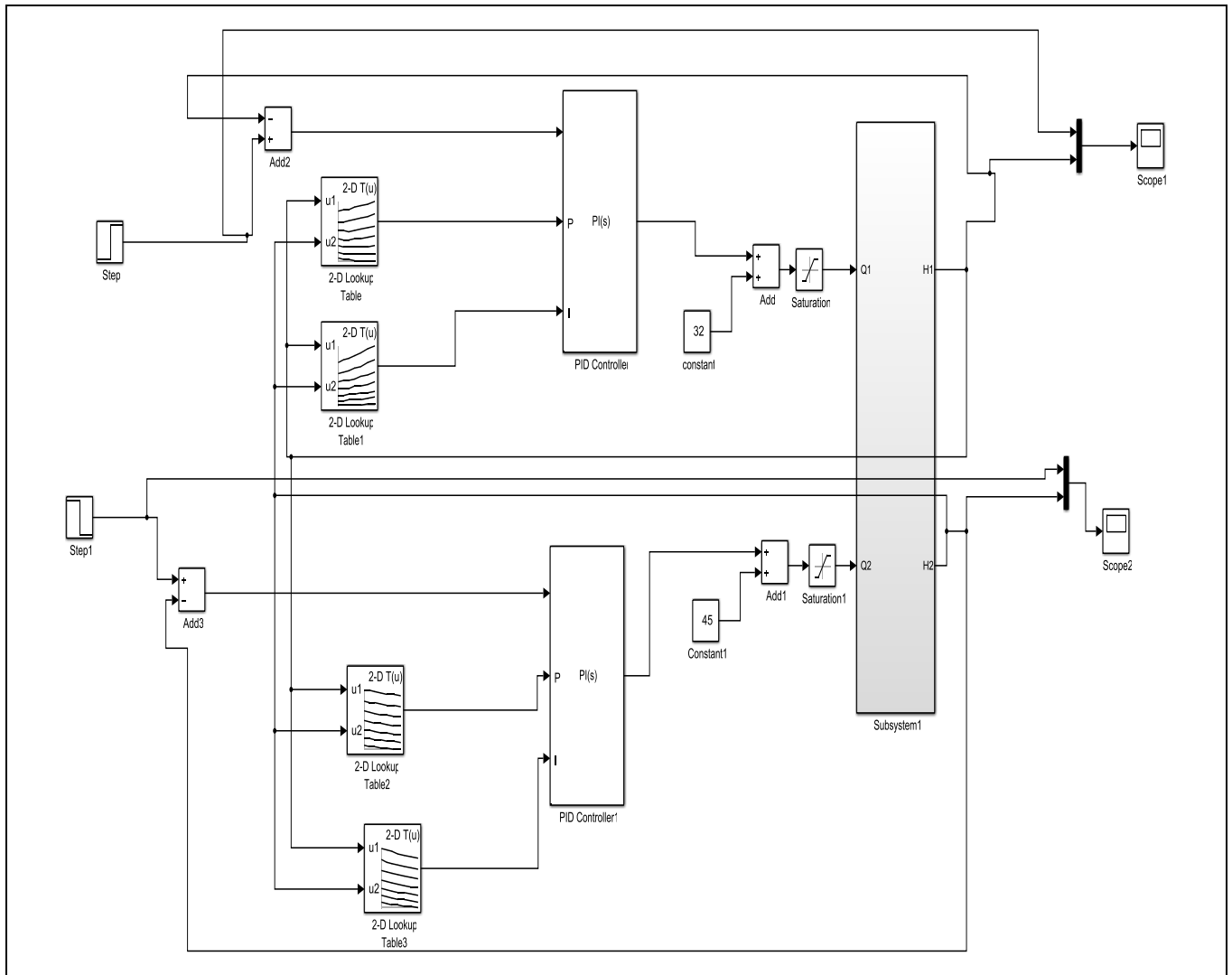


Figure 2: The Simulink diagram showing the gain scheduling adaptive control configuration

The following are the results obtained for different step inputs within the operating range of the three tank system.

1. For step input into the system ranging from 37.749 to 42.310, the response is shown below:

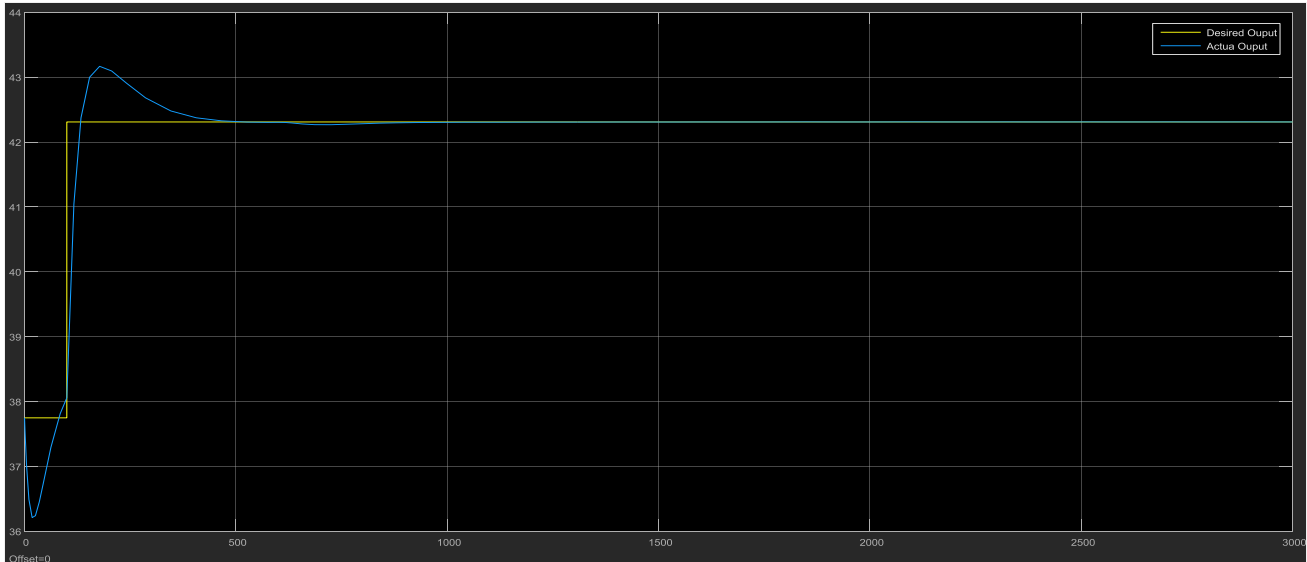


Figure 3: The Desired output(step) versus the Actual output for step input (37.749-42.310).

- For step input ranging from 16.800 to 15.145 within the operating range of the system, the response obtained is shown below:

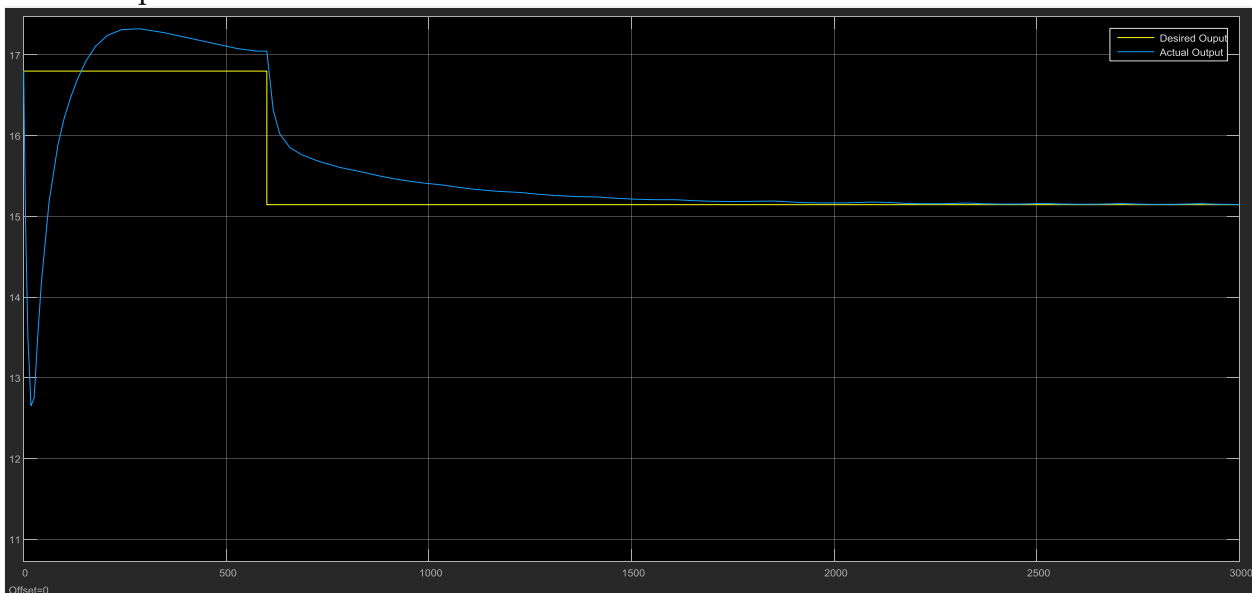


Figure 4: The Desired Output versus Actual Output for step input (16.800-15.145).

V. FUTURE RESEARCH

- Improve controller design methods to achieve better reference tracking.
- The current controller design is insufficient for some input ranges; adopting more effective tuning methods is necessary

VI. CHALLENGES

The main challenges encountered include difficulty in achieving accurate reference tracking across all operating ranges, particularly for certain input variations. Additionally, linearizing the nonlinear three-tank system posed complexities in maintaining consistent performance. Tuning the PI controllers also required extensive trial and error to optimize control parameters effectively.

VII. CONCLUSION

1. A nonlinear three-tank system with three states, two inputs, and two outputs was considered.
2. The system was linearized at seven different operating points, resulting in seven linear models.
3. PI controllers were designed using Cohen-Coon, Ziegler-Nichols, and Chien-Hrones-Reswick tuning rules.
4. Cohen-Coon tuning rule was adopted for lower error.
5. PI controllers were developed for diagonal terms, resulting in 49 controllers in total.
6. Controller parameters adjusted well for step input ranges from 37.749 to 42.310 but showed poor reference tracking for the range 16.800 to 15.145.

REFERENCES

1. P. Ioannou and B. Fidan (2006). Adaptive control tutorial. Society for industrial and Applied Mathematics, Philadelphia.
2. Wang, M. and Crusca, F. (2002). "Design and Implementation of a Gain Scheduling Controller for a Water Level Control System." ISA Transaction, 41, 323-331. DOI: [10.1016/S0019-0578(07)60091-3]([https://doi.org/10.1016/S0019-0578\(07\)60091-3](https://doi.org/10.1016/S0019-0578(07)60091-3)).
3. Liu Q., Chai T., Zhao L., Zhang Y., and Zhang S. (2009). Modeling and Gain Scheduling Adaptive Control of Tension Control System for Continuous Annealing Process. Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference. ThC10.5 5599-5603.
4. Jang J., Annaswamy A M., and Lavretsky E. (2008). Adaptive control of time-varying systems with gain scheduling. 2008 American Control Conference. ThC14.5. 3416-3421
5. Sharhruz S.M., and Behtash S. (1992). Design of Controllers for Linear Parameter-Varying Systems by the Gain Scheduling Technique. Journal of Mathematical Analysis and Applications. 168. 195-217
6. Zavari K., Pipeleers G., and Swevers J. (2013). Industrial Electronics, IEEE Transactions. 61:7, 3713-3718
7. Gnoth, S., Kuprijanov, A., Simutis, R., & Lübbert, A. (2010). "Simple adaptive pH control in bioreactors using gain-scheduling methods." Applied Microbiology and Biotechnology, 85, 955-964. DOI: [10.1007/s00253-009-2114-5](<https://doi.org/10.1007/s00253-009-2114-5>).
8. Paijmans B., Symen W., Brussel H.V., and Swevers J. (2006). Gain scheduling control for mechatronic systems with position dependent dynamics. Proceedings of ISMA 2006

9. Fujimori A.,Gunnarsson S., and Norrlof M. (2000). A gain scheduling control of nonlinear systems along a reference trajectory. Division of automatic control, Department of Electrical Engineering, Linkopings universitet, SE-581 83 Linkoping, Sweden.
10. Naik M. N.,Desiree C.M., and Monterio S.N. (2015). Design and Simulation of Gain Scheduled Adaptive Controller using PI controller for Conical Tank Process. International Journal for Innovative Research in Science &Technology,2:4,2349-6010
11. Pakmehr M., Fitzgerald N.,Feron E M.,Shamma J S., andBehbahani A. (2013). Gain Scheduling control of gas turbine engines. Proceedings of ASME Turbo Expo 2013.GT2013-96012.